

Higher-Order Forward Guidance

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Presentation Slides

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- “*The Federal Reserve, . . . affirmed today its **readiness to serve as a source of liquidity to support the economic and financial system***” - Greenspan, 1987 (Black Monday)
- “*Within our mandate, the ECB is **ready to do whatever it takes to preserve the euro. And believe me, it will be enough.***”- Draghi, 2012 (Euro-Crisis)

Big Question (Uncertainty Management)

How to manage economic uncertainty? Is it possible? Desirable?

- 1 Unconventional policy interventions (e.g. forward guidance) becoming more prevalent
- 2 Uncertainty is an important source of Business Cycle fluctuations
 - Bloom (2009), Ludvigson et al. (2015),...
 - Finance: risk-premium \propto volatility² (e.g., Merton (1971))
 - **VAR analysis**: financial and real volatility ▶ VAR analysis
- 3 Uncertainty as a coordination failure (sometimes)
- 4 **This paper**: Forward Guidance with a focus on strategic uncertainty management and coordination

Non-linear New-Keynesian model with a stock market + portfolio

1. **Build** a parsimonious New-Keynesian framework where: [» Explain](#)

Stock volatility↑ \iff risk-premium↑ \iff wealth↓ \iff aggregate demand↓

- Asset price as endogenous shifter in aggregate demand (and vice-versa)
 - Multiplicity of intertemporal equilibria (via agent coordination problem)
2. Study several Forward Guidance interventions, from traditional to 'Higher-order'
 - Monetary policy
 - Fiscal policy (in progress)
 3. Findings: New trade-off between current and future financial stability

Identical **capitalists** and **hand-to-mouth workers** (Two types of agents)

- **Capitalists:** consumption - portfolio decision (between stock and bond)
- **Workers:** supply labors to firms (hand-to-mouth)

Fundamental risk
(Exogenous)

1. Technology

$$\frac{dA_t}{A_t} = \underbrace{g}_{\text{Growth}} \cdot dt + \sigma \cdot \underbrace{dZ_t}_{\text{Aggregate shock}}$$

2. Hand-to-mouth workers: supply labor + solves the following problem

$$\max_{C_t^w, N_t^w} \frac{\left(\frac{C_t^w}{A_t}\right)^{1-\varphi}}{1-\varphi} - \frac{(N_t^w)^{1+\chi_0}}{1+\chi_0} \quad \text{s.t.} \quad \bar{p}C_t^w = w_t N_t^w$$

- Hand-to-mouth assumption can be relaxed, without changing implications

3. Firms: Dixit-Stiglitz production using labor + perfectly rigid prices ($\pi_t = 0$)

4. Financial market: zero net-supplied risk-free bond + stock (index) market

Capitalists: standard portfolio and consumption decisions (very simple)

1. Total financial wealth $a_t = \bar{p}A_tQ_t$, where (real) stock price Q_t follows:

$$\frac{dQ_t}{Q_t} = \mu_t^q \cdot dt + \sigma_t^q \cdot dZ_t$$

Financial risk
(Endogenous)

- μ_t^q and σ_t^q are both endogenous (to be determined)

2. Each solves the following optimization (standard)

$$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} \log C_t dt \quad \text{s.t.}$$

$$da_t = (a_t(i_t + \theta_t(i_t^m - i_t)) - \bar{p}C_t)dt + \theta_t a_t (\sigma + \sigma_t^q) dZ_t$$

- Aggregate consumption of capitalists \propto aggregate financial wealth

$$C_t = \rho A_t Q_t$$

- Equilibrium **risk-premium** is determined by the total risk

$$i_t^m - i_t \equiv rp_t = (\sigma + \sigma_t^q)^2$$

Dividend yield: dividend yield = ρ , as in Caballero and Simsek (2020)

- A positive feedback loop between asset price \iff dividend (output)

Determination of nominal stock return dI_t^m

$$dI_t^m = \left[\underbrace{\rho}_{\text{Dividend yield}} + \underbrace{g + \mu_t^q + \overbrace{\sigma\sigma_t^q}^{\text{Covariance}}}_{\text{Capital gain}} \right] dt + \underbrace{(\sigma + \sigma_t^q)}_{\text{Risk term}} dZ_t$$

$$= \underbrace{i_t^m}_{\text{Drift}} = \underbrace{i_t}_{\text{Monetary policy}} + \underbrace{(\sigma + \sigma_t^q)^2}_{\text{Risk-premium}}$$

Flexible price economy as benchmark: the 'natural' consumption of capitalists $C_t^n = \rho A_t Q_t^n$ follows

$$\begin{aligned} \frac{dC_t^n}{C_t^n} &\equiv \frac{d(A_t Q_t^n)}{A_t Q_t^n} = (r^n - \rho + \sigma^2) dt + \sigma dZ_t \\ &= g dt + \sigma dZ_t = \frac{dA_t}{A_t} \end{aligned}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

Define **asset price gap**

$$\hat{Q}_t = \ln \frac{Q_t}{Q_t^n}, \quad \underbrace{0 = \text{Var}_t \left(\frac{dQ_t^n}{Q_t^n} \right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\overset{\text{Endogenous}}{\sigma_t^q} \right)^2 dt = \text{Var}_t \left(\frac{dQ_t}{Q_t} \right)}_{\text{Actual volatility}}$$

which is proportional to **output gap**

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \implies \hat{Y}_t = \underbrace{\zeta}_{>0} \cdot \hat{Q}_t$$

A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Q}_t = \left(i_t - \underbrace{\left(r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2 \right)}_{\equiv r_t^T} \right) dt + \sigma_t^q dZ_t \quad (1)$$

New terms
↗ ↘

- What is r_t^T ?: a **risk-adjusted** natural rate of interest ($\sigma_t^q \uparrow \implies r_t^T \downarrow$)

$$\begin{aligned} r_t^T &\equiv r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2 \\ &= r^n - \hat{r}p_t \end{aligned}$$

where $\underbrace{\hat{r}p_t = rp_t - \hat{r}p_t^n}_{\text{risk-premia gap}}$.

Big Question: Taylor rule $i_t = r^n + \phi_q \hat{Q}_t$ for $\phi > 0 \Rightarrow$ **full stabilization?**

Up to a first-order (no volatility feedback): **Blanchard and Kahn (1980)**

- $\phi_q > 0$: Taylor principle $\implies \hat{Q}_t = 0$ for $\forall t$ (unique equilibrium)
- **Why?** (recap): without the volatility feedback:

$$d\hat{Q}_t = (i_t - r^n) dt + \sigma_t^q dZ_t \quad \underbrace{=}_{\substack{\text{Under} \\ \text{Taylor rule}}} \quad \phi_q \hat{Q}_t dt + \sigma_t^q dZ_t$$

Then,

$$\mathbb{E}_t(d\hat{Q}_t) = \phi_q \hat{Q}_t$$

- If $\hat{Q}_t \neq 0$, then $\mathbb{E}_t(\hat{Q}_\infty)$ blows up $\rightarrow \hat{Q}_t = 0$ for $\forall t$ as unique equilibrium
- Foundation of modern central banking

A modified monetary rule: targeting of risk-premium

New monetary policy \implies financial + macro stability $\hat{Y}_t = \hat{Q}_t = \hat{r}p_t = 0$

$$i_t = r^n + \phi_q \hat{Q}_t - \overbrace{\frac{1}{2} \hat{r}p_t}^{\text{New targeting}}, \text{ restores determinacy.}$$

Leading to:

$$\underbrace{i_t + \hat{r}p_t - \frac{1}{2} \hat{r}p_t}_{= i_t^m} = \underbrace{r^n + \hat{r}p^n - \frac{1}{2} \hat{r}p^n}_{= i_t^{m,n}} + \underbrace{\phi_\pi \pi_t + \phi_q \hat{Q}_t}_{\text{Business cycle targeting}}$$

$\rho + \frac{\mathbb{E}_t(d \log a_t)}{dt}$
 $\rho + \frac{\mathbb{E}_t(d \log a_t^n)}{dt}$

Ito term
Ito term

- The (expected) return on aggregate wealth, not just the policy rate, must follow a Taylor rule

Thought experiment: fundamental volatility $\sigma \uparrow$: from $\underline{\sigma}$ to $\bar{\sigma}$ on $[0, T]$ (e.g., [Werning \(2012\)](#)) and comes back to $\underline{\sigma}$ with

- $r_1^n \equiv \rho + g - \underline{\sigma}^2 > 0$: no ZLB before
- $r_2^n \equiv \rho + g - \bar{\sigma}^2 < 0$: now ZLB binds (on the stabilized equilibrium path)

Assume: perfect stabilization (i.e., $\hat{Q}_t = 0$) is achievable outside ZLB

- Central bank always can use risk-premium targeting as given by

$$i_t = r_1^n + \phi_q \hat{Q}_t - \frac{1}{2} \hat{p}_t, \quad \text{with } \phi_q > 0$$

Result: perfect stabilization inside ZLB as well

- **Recursive argument:** Full stabilization at T implies stabilization at $T - dt$, and so on...

ZLB path (full stabilization after T)

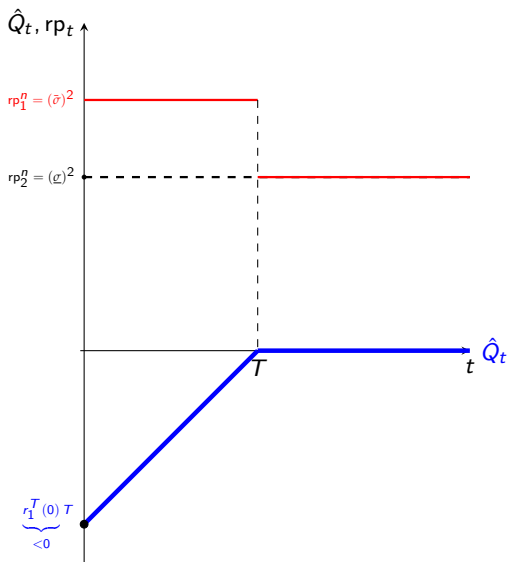


Figure: ZLB dynamics (Benchmark)

Assume:

- Central bank can commit to keep $i_t = 0$ until $\hat{T} \geq T$
- Perfect stabilization (i.e., $\hat{Q}_t = 0$) afterwards, $t > \hat{T}$
 - By previous: also stabilization beforehand, $t \leq \hat{T}$

Problem: Minimize smooth quadratic welfare loss

$$\min_{\hat{T}} \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} (\hat{Q}_t)^2 dt$$

$$\text{s.t. } \hat{Q}_0 = \underbrace{r_1^T (\sigma_1^q = 0)}_{<0} T + \underbrace{r_2^T (\sigma_2^q = 0)}_{>0} (\hat{T} - T)$$

where restriction follows from the non-linear IS equation

Traditional forward guidance (keep $i_t = 0$ until $\hat{T} > T$)

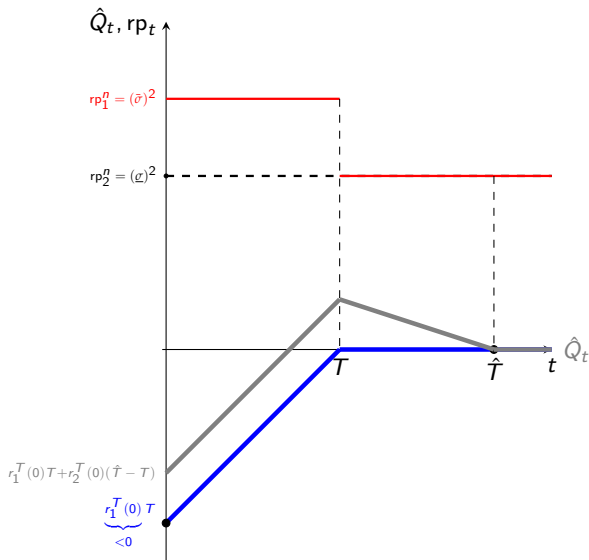


Figure: ZLB dynamics with forward guidance until $\hat{T} > T$

Recall an economic mechanism in the ZLB and forward guidance

1. Central bank achieves perfect stabilization: $\hat{Q}_t = \hat{r}p_t = 0, \forall t \geq \hat{T}$



2. $\hat{Q}_{\hat{T}} = 0$ guarantees $\sigma_t^q = \sigma^{q,n} = 0, rp_t = rp^n$ for $t \leq \hat{T}$

Still if rp^n is too high, might want to push $\{\sigma_t^q, rp_t\}$ down for $\hat{Q}_t \uparrow$?

- Thus achieve $\sigma_t^q < \sigma^{q,n} = 0, rp_t < rp^n \implies \hat{Q}_t \uparrow$ at the ZLB

Take **contrapositive** to the above:

$\neg 2. \sigma_t^q < \sigma^{q,n} = 0, rp_t < rp^n$ for $t \leq \hat{T}$



$\neg 1. \hat{Q}_{\hat{T}} \neq 0$. Central bank commits not to perfectly stabilize the economy after \hat{T}

- Giving up **future** financial stability $\implies rp_t \downarrow$ and $\hat{Q}_t \uparrow$ **now** (at the ZLB)

Assume:

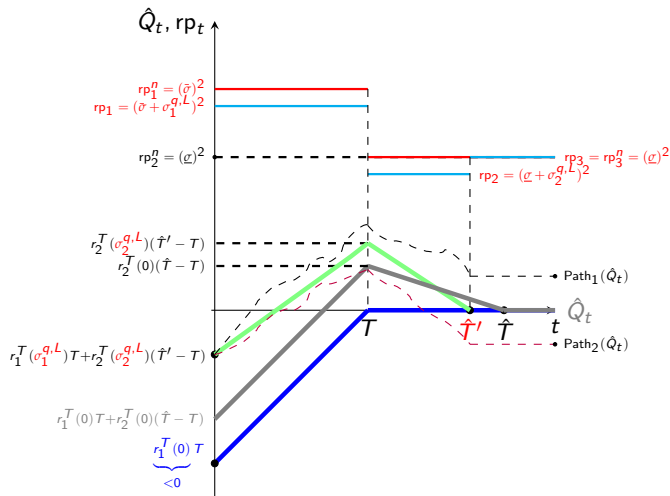
- Central bank can commit to keep $i_t = 0$ until $\hat{T}' \geq T$
- No asset price gap stabilization (i.e., $\hat{Q}_t = \hat{Q}_{\hat{T}'}$) afterwards, $t \geq \hat{T}'$
- Pick $\{\sigma_t^q\}$ for $t < \hat{T}'$

Problem: Minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}'} \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} (\hat{Q}_t)^2 dt,$$

$$\text{s.t. } \begin{cases} d\hat{Q}_t = - \underbrace{r_1^T (\sigma_1^{q,L})}_{<0} dt + \sigma_1^{q,L} dZ_t, & \text{for } t < T, \\ d\hat{Q}_t = - \underbrace{r_2^T (\sigma_2^{q,L})}_{>0} dt + \sigma_2^{q,L} dZ_t, & \text{for } T \leq t < \hat{T}', \\ d\hat{Q}_t = 0, & \text{for } t \geq \hat{T}', \end{cases}$$

$$\hat{Q}_0 = \underbrace{r_1^T (\sigma_1^{q,L})}_{<0} T + \underbrace{r_2^T (\sigma_2^{q,L})}_{>0} (\hat{T}' - T)$$

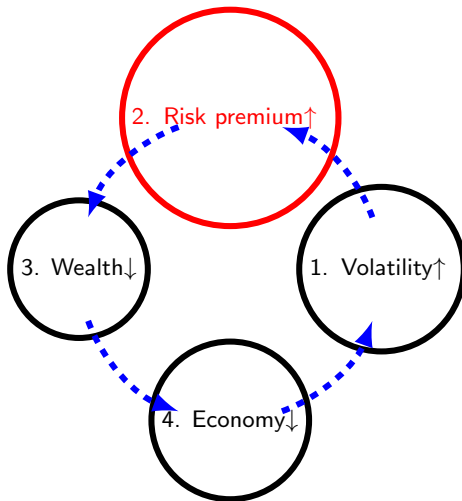


Proposition (Optimal commitment path)

At optimum, $\sigma_1^{q,L} < \sigma_1^{q,n}$, $\sigma_2^{q,L} < \sigma_2^{q,n}$, and $\hat{T}' < \hat{T}$

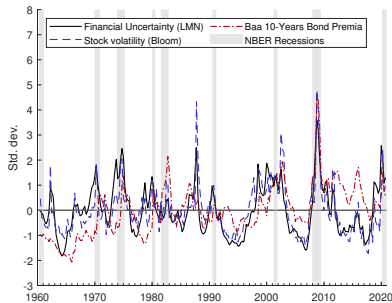
- Traditional forward guidance is good, but can do better
- Trade-off between current and future financial stability
- Exercise: Extreme but simple
 - Instead: stabilization with ν probability after forward guidance period ends
- Central bank credibility still a necessary condition

Thank you very much!
(Appendix)

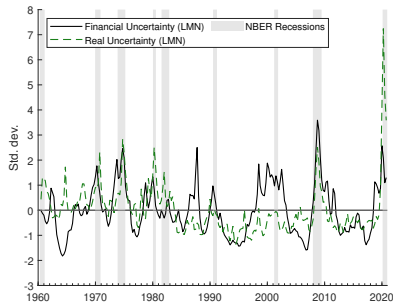


- 1 → 2 comes from “non-linearity (not linearizing)”
- 2 → 3 comes from “portfolio decision” of each investor and externality
- 3 → 4 comes from the fact wealth drives aggregate demand
- 4 → 1 where business cycle has its own volatility (self-sustaining)

Go back



(a) Financial Uncertainty series



(b) Financial vs. Real Uncertainty

Figure: Common measures of the financial volatility (left) and real vs. financial uncertainty decomposed by Ludvigson et al. (2015) (right)

The correlation between series is remarkably high and they all display positive spikes at the beginning and/or initial months following NBER-dated recessions

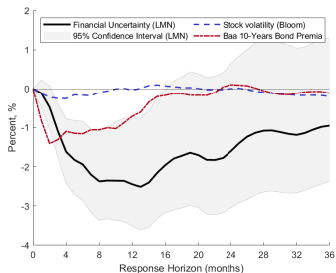
- Many of past recessions are, in nature, financial

In a similar manner to Bloom (2009), Ludvigson et al. (2015):

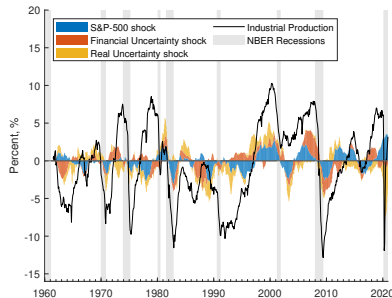
$$\text{VAR-11 order:} \quad \left[\begin{array}{c} \log(\text{Industrial Production}) \\ \log(\text{Employment}) \\ \log(\text{Real Consumption}) \\ \log(\text{CPI}) \\ \log(\text{Wages}) \\ \text{Hours} \\ \text{Real Uncertainty (LMN)} \\ \text{Fed Funds Rate} \\ \log(\text{M2}) \\ \log(\text{S\&P-500 Index}) \\ \text{Financial Uncertainty (LMN)} \end{array} \right] \quad (2)$$

Financial uncertainty (LMN) is also replaced by the stock price volatility (following Bloom (2009)) and Baa 10-years bond premia

Vector Autoregression (VAR) analysis



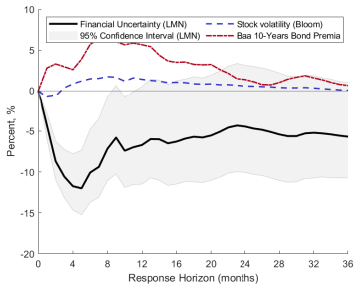
(a) Response: Industrial Production



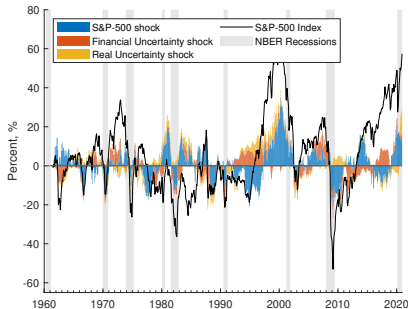
(b) Industrial Production

Figure: Impulse-response of IP to one std.dev shock in financial uncertainty measures (left) and the historical decomposition of IP to various attributes (right)

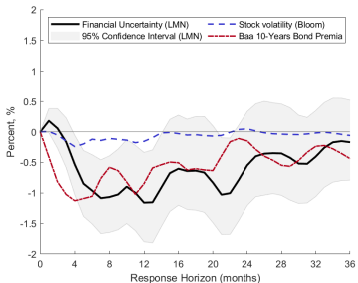
- 1 IP falls by 2.5% after one standard deviation spike in the Ludvigson et al. (2015)'s financial uncertainty measure
 - Financial uncertainty has been important in driving IP boom-bust patterns
- 2 Other graphs: IRF and historical decomposition of S&P 500 [▶ S&P500](#), and FFR (monetary policy) [▶ FFR](#), FEVD [▶ FEVD](#)



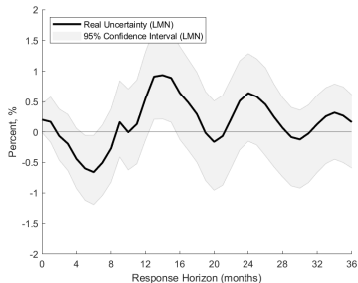
(a) Response: S&P-500 Index



(b) S&P-500 Index



(a) Shock: Financial Uncertainty



(b) Shock: Real Uncertainty

With 3 different financial uncertainty measures: Ludvigson et al. (2015), Bloom (2009), Baa 10-years bond premia (left)

(i) Industrial Production

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0	0.30	0.21	0.12
h=6	1.27	3.37	2.98	1.36
h=12	4.28	4.38	3.16	1.94
h=36	3.24	1.67	1.98	0.64

(ii) S&P-500 Index

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0.11	0.08	0.39	0.06
h=6	3.30	0.25	3.26	0.62
h=12	4.77	0.54	10.03	2.16
h=36	6.50	0.91	12.16	2.40

(iii) Fed Funds Rate

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0.01	0.98	0	0.08
h=6	0.42	0.84	3.11	1.66
h=12	1.47	0.91	4.69	2.30
h=36	2.81	2.05	5.02	3.17

Financial uncertainty shocks explain close to:

- 5% of the fluctuations in both IP and S&P-500 series

Real uncertainty explains:

- Additional 2-4% of movements in industrial activity in the medium run