Efficiency, Risk and the Gains from Trade in Interbank Markets *

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Abstract

Bank-to-bank markets play a central role in efficient liquidity provision. However, by propagating granular shocks between banks, they may also be a source of aggregate risk. In this paper, we develop a quantitative trade framework of the interbank market and embed it into a DSGE model to capture the trade-off between efficiency and risk accompanying interbank market integration. In the model, we derive analytical approximations for welfare that depend on features of the interbank network and a few key elasticities. Using microdata on bilateral asset positions for the population of German banks, we estimate the key elasticities with plausibly exogenous variation in banks' exposure to the US financial crisis via interbank connections. Our findings indicate that the current level of interbank market integration improves welfare by 1.33%, while active provision of credit to distressed banks by the lender-of-last-resort reduces the welfare costs of idiosyncratic short-term financial shocks.

Keywords: Bank Networks; Interbank Markets; Trade; Lender-of-last-resort

JEL Codes: E32, E44, E51, F19, G21

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1 Introduction

The financial crisis of 2007/2008 and the subsequent global recession highlighted the devastating effects of financial shocks propagating to the real economy. The crisis spread through the entire financial system in the US and abroad when a few large financial institutions, primarily Lehman Brothers, failed to service their debt obligations with other banks. Indeed, the level of interbank market integration – the degree to which banks are interconnected by bilateral lending and borrowing – emerges as a key determinant of whether shocks to large, "granular" banks lead to *contagion risk* in the financial sector as a whole.

At the same time, interbank market integration also offers benefits. In the presence of small idiosyncratic funding shocks, interbank relationships enable banks to substitute between funding sources, resulting in less volatile funding costs and, consequently, dynamic gains due to *risk diversification*. In addition to this dynamic trade-off, market integration is accompanied by improvements in liquidity provision to banks, thereby lowering banks' funding costs and leading to static *efficiency gains* in terms of output and welfare.

Overall, the net effect of market integration on aggregate volatility and welfare is ambiguous, and its size and direction depend on salient market features, such as bank heterogeneity, market concentration, and the degree of participation. However, realistic interbank markets are largely absent from quantitative macroeconomic models, limiting their ability to address this question. Conversely, network models of financial intermediation seldom incorporate the analysis of business cycles to study the welfare implications of interbank markets.

In this paper, we develop a framework that can account for a realistic interbank network while remaining tractable enough to study the implications of market integration for aggregate volatility and welfare. Specifically, drawing on recent models in international trade, our interbank network retains convenient aggregation properties despite featuring a large number of heterogeneous banks, funds as a homogeneous good, and various shocks. To address aggregate welfare, we embed the network into a New Keynesian DSGE model and derive an analytical approximation for the welfare gains from interbank trade that holds under arbitrary network structures and requires only a small set of elasticities. Using the welfare formula, we flexibly study the gains from interbank trade under various scenarios, such as a given level of integration, counterfactual integration levels, or changing network structures.

Moreover, our model offers an analytical structure for examining the liquidity provision strategies employed by central banks in their capacity as the economy's lender-of-last-resort. This entails the proactive extension of credit to financially troubled banks, consequently diminishing financial market volatility by imposing a limit on banks' funding expenses. This framework allows us to evaluate the welfare consequences of various lender-of-last-resort approaches, including the countercyclical provision of liquidity or the prioritization of supporting certain banks, such as those considered "too big to fail".

In our model, a representative household may save through bonds and derives utility from holding real deposit balances across a discrete array of banks. However, within each period, her relative preferences concerning banks are subject to rapid, transient shocks. Banks transfer these savings to firms, which require a consistent supply of bank loans to finance capital investments essential for production throughout each period. This feature generates brief liquidity mismatches, subsequently motivating banks to trade homogeneous funds in the interbank market, subject to transaction costs. Consequently, our model can accommodate typical features of interbank markets, such as substantial gross debt positions between banks and structural net positions, without presuming that banks trade in differentiated goods.[1] Across periods, banks encounter fluctuating funding costs due to idiosyncratic and potentially correlated shocks to transaction costs, loan demand, and deposit supply. Depending on market integration and structure, such shocks can contribute to volatility in the economy-wide interest rate spread over the bond rate, which constitutes a crucial aspect of the model's DSGE component. The remaining features of the DSGE component are relatively standard (Calvo pricing, Taylor rule, etc.), resulting in a familiar dynamic system of equations. The primary distinction is that the natural interest rate relies on the interbank market spread over the bond rate, and its volatility influences the output gap, ultimately affecting aggregate volatility and welfare.

At the core of our analysis lies the concept of gains from trade, which, in the context of our paper, refers to the static and dynamic welfare benefits or costs associated with varying degrees of interbank market integration.[2] By deriving an analytical approximation for welfare, we establish a formula that quantifies gains from trade in stochastic environments, capturing the balance between efficiency and risk intrinsic to financial integration processes. We demonstrate that observable moments in the data, such as the degree of market integration, bank-level Herfindahl indices for funding sources, and the stochastic properties of idiosyncratic shocks, operate as sufficient statistics for welfare gains.

We demonstrate the applicability of our model by examining the gains from interbank trade and the central bank's role in the context of the German banking sector. To achieve this, we construct a database utilizing proprietary microdata from the Deutsche Bundesbank, which contains information on individual balance sheets and bilateral asset/debt positions for all active banks in Germany. Our assessment of welfare gains is fundamentally dependent on the structure of the interbank market, which we investigate in the context of the German financial market. First, a high concentration at the top of the size distribution suggests that shocks to large individual banks may cause contagion in the market and propel aggregate economic fluctuations (*contagion risk*). Second, the German bank-to-bank market volume is substantial, with interbank liabilities constituting 29% of banks' balance sheets prior to 2007/08 and experienc-

[1] We draw upon Farrokhi (2020), who formulates a trade model encompassing homogeneous goods and bidirectional trade.

[2] Arkolakis, Costinot, and Rodríguez-Clare (2012) explore the gains from trade in the context of international trade. They demonstrate that the observed level of market openness functions as a sufficient statistic for the welfare gains from trade relative to autarky across a wide range of static trade models. Our paper not only replicates their insights regarding static welfare gains from trading funds but also extends their findings to encompass stochastic environments.

ing a 6 percentage point decline following the Great Recession. Our welfare formula establishes a direct connection between welfare gains and market integration, indicating potential welfare losses due to the reduction in market volume after 2008. Third, the interbank market displays a core-periphery structure, characterized by a small core of large banks with 100 or more connections and a periphery of smaller banks with no more than 10 interbank partners.[3] In the context of idiosyncratic liquidity shocks, the core-periphery structure restricts many banks' ability to substitute funding sources (*risk diversification*), while concurrently exposing the market to *contagion risk* from a limited group of large banks situated at the network's center.

The impact of the 2007/08 US financial crisis on the German interbank market provides a prime example of a contagion event. We exploit the crisis as a natural experiment to present causal evidence for the significance of interbank markets in relation to banks' funding and lending, as well as to test our model's predictions. Several large German banks suffered substantial losses in their US asset holdings and consequently reduced lending to domestic partner banks. Viewed through the lens of our model, such indirectly exposed borrowers encounter elevated funding costs, as liquidity must be financed through potentially more expensive means, such as deposits or equity. To formalize this mechanism, we devise a bank-level measure of indirect exposure to the crisis by interacting a bank's liabilities vis-à-vis directly exposed domestic lenders with the latter's level of US bank asset holdings prior to 2008. Following the turmoil of 2007/08, more affected banks increased interest rates on non-financial loans by approximately 20 basis points on average due to higher funding costs, and reduced lending to firms and consumers by up to 5%. Our findings indicate that banks with high indirect exposure indeed significantly decreased interbank borrowing, and financed a larger share of non-financial loans from their own sources, such as deposits and equity (by around 3.5%). In order to credibly quantify the gains from trade, we estimate the key demand and supply elasticities of our model utilizing the funding cost shock as plausibly exogenous variation in interest rates.

Within our framework, interbank trade is subject to transaction costs, which we model as stochastic exogenous "wedges".[4] We remain neutral regarding the origin of these wedges; instead, we derive them directly from the data as structural residuals for each quarter, interbank connection, and bank.[5] To achieve this, we reformulate the equilibrium relationships of the interbank market model such that the wedges are functions of the data. Combined with our estimated elasticities from the previous step, we extract the set of wedges, ensuring that the model reproduces the data exactly in each quarter. Subsequently, we compute the stochastic properties of the recovered wedges as components of our welfare formula. In the absence of wedges in interbank trade, the model approaches the free trade benchmark characterized by zero intermediation costs and minimal volatility, whereas infinite wedges result in financial autarky,

[3]See Craig and Ma (2022) for additional details concerning the core-periphery structure of the German interbank market.

[4]The other exogenous and volatile parameters of the model are deposit supply and loan demand shocks.

[5]Possible micro-foundations for such wedges proposed in the literature include asymmetric information (e.g., Babus and Hu (2017), Babus and Kondor (2018)), costly link formation (e.g., Craig and Ma (2022)), and intermediation spreads (e.g., Farboodi (2021)).

wherein banks fund loans exclusively through volatile deposits, thereby increasing funding costs but avoiding interbank market volatility. Our recovered transaction costs lie between these extremes, giving rise to an efficiency-volatility trade-off in welfare that is central to our paper.

Utilizing these parameters and estimated model elasticities, we employ our analytical expressions to compute the welfare gains of financial market integration under various levels of transaction costs. Our findings indicate that the current levels of interbank market integration contribute positively to welfare on net by 1.33% of quarterly consumption through efficiency gains and by mitigating volatility via diversification, which in practice outweighs the costs of contagion risk. The Great Recession and the European Bond crisis persistently diminished participation in interbank markets and elevated credit spreads. Our model establishes a link between these two events, and we estimate utility losses stemming from the reduction in interbank market activity to be approximately 0.56% of quarterly consumption.

In the last step of our analysis, we enable central banks to directly inject funds into the market as a lender-of-last-resort, thereby imposing a limit on the costs of funds. We disentangle the welfare effects of such interventions into two distinct components. First, liquidity provision by central banks has the capacity to mitigate short-term idiosyncratic shocks to the funding costs of banks. Second, lender-of-last-resort policies can also address the welfare losses stemming from cyclical fluctuations in interbank funding. We estimate that the presence of such a discount window improves welfare by 2.5% of consumption per quarter, with the majority of the gains arising through the first channel, while a provision of funds sensitive to the cycle has limited impact on welfare gains. Finally, we discover that the benefits derived from the presence of a lender-of-last-resort are disproportionately attributed to the discount window access enjoyed by the larger and better-connected banks of the system, which aligns with their centrality in the interbank market.

Following earlier work by Bernanke and Gertler (1986), Kiyotaki and Moore (1997), and the financial accelerator of Bernanke, Gertler, and Gilchrist (1998), a substantial literature emerged to study how the financial system, in its role as an intermediary between household savings and firms' investment, generates credit frictions that amplify business cycle fluctuations. While the majority of these papers concentrate on the bank-to-firms or depositor-to-banks aspects of the financial channel, others, such as Cingano, Manaresi, and Sette (2016), empirically demonstrate that a significant portion of the decline in bank lending observed during the Great Recession can be attributed to the freezing of interbank markets. Our paper offers a theoretical framework to analyze these channels jointly.

However, from a theoretical standpoint, devising a business cycle model that accurately captures the stylized facts surrounding bank-to-bank markets while maintaining analytical tractability is highly challenging. Among the few papers that investigate the impact of the interbank market on the macroeconomy, compromises are made in favor of tractability: Gertler, Kiyotaki, and Prestipino (2016) simplify the problem by assuming two types of banks, retail banks that obtain deposits from households and wholesale banks that borrow from retail banks. Piazzesi, Rogers, and Schneider (2019) and De Fiore, Hoerova, and Uhlig (2018) build on search models that presume a continuum of atomistic banks differentiated solely by the magnitude of the liquidity shock they receive each period. In this paper, we propose an alternative approach by adapting the Eaton and Kortum (2002) model of international trade to the banking sector. Trade models are naturally well-suited for this task, as they often feature a discrete number of heterogeneous agents and straightforward expressions for trade volumes and cost structures.

Hence, our paper relates to research in international finance and macroeconomics that studies financial markets with imperfect substitution in asset demand. Koijen and Yogo (2020) solve for bilateral asset holdings across countries using an asset demand system that takes a logit-form similar to our interbank model. Kleinman, Liu, Redding, and Yogo (2023) propose a quantitative, open-economy growth model in which assets are subject to extreme valued-distributed productivity shocks leading to imperfect substitution in asset demand. Our interbank framework shares this property, however, we propose a micro-foundation for imperfect substitution in funds demand based on short-term liquidity mismatches.

Drawing on earlier theoretical work by Allen and Gale (2000) and more recently Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), another extensive literature on banking networks delineates the conditions under which interbank markets emerge and yield a trade-off between an efficient allocation of funds and an increased risk of contagion (or default, volatility, etc.).[6] Craig and Ma (2022) develop a network model of financial intermediation based on data from the German banking system. However, the focus of this literature remains on the banking system itself. Instead of attempting to explain why the interbank market developed its current structure, we accept it as given and explore how this structure contributes to the efficiency and volatility of the economy, as well as how lender-of-last-resort policy can alleviate its adverse effects on welfare.

The remainder of this paper is structured as follows. Section 2 introduces the model. Section 3 derives the welfare approximation to market integration and lender-of-last-resort intervention. Section 4 presents the data. Section 5 highlights some key aspects of the German banking system. Section 6 empirically estimates the effects of the 2007/08 financial crisis on the German interbank market and calibrates the model. Section 7 discusses the quantification exercises. Finally, Section 8 offers concluding remarks.

2 Model

Our setting consists of a standard New-Keynesian economy augmented with a banking sector composed of N distinct banks. Figure 1 depicts the different components of the model. Funds enter the banking system via household savings in the form of deposits and are forwarded to firms, which borrow to build capital stock for production. Mismatches between loan demand and deposits give rise to the interbank market, as well as a rationale for the existence of a central bank's lending facility to provide emergency credit to the system. A detailed model derivation

[6] See, for example, Farboodi (2021), Babus and Hu (2017), Babus and Kondor (2018), Chang and Zhang (2021), Üslü (2019)

can be found in Appendix 1.

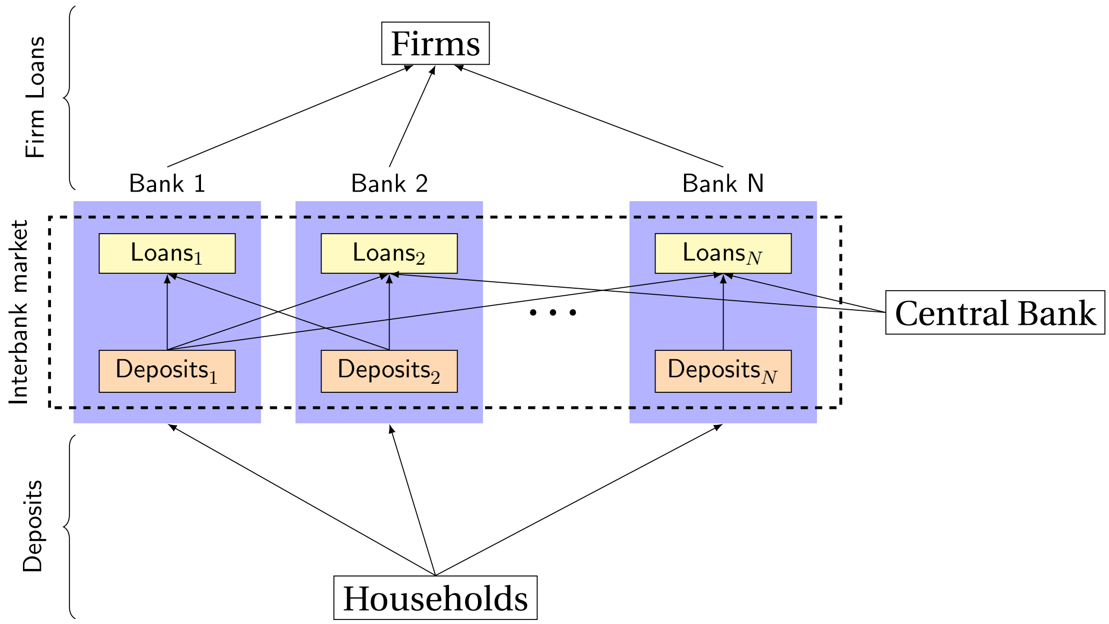

Figure 1: Components of the financial channel: Households allocate savings across banks in the form of deposits. Banks lend these funds to firms, which utilize them to finance capital investments. Mismatches between bank deposits and firm loan demand are resolved in the interbank market. A central bank supplies lender-of-last-resort (LoLR) credit to the system banks. Arrows indicate the flow of funds between agents.

2.1 Notation and Timing Conventions

The model features discrete quarters indexed by t, and each quarter is divided into a $[0, 1]$ continuum[7] in which agents take actions, such as consumption, employment, or saving decisions. A moment within the continuum is indexed by τ, and the pair $\{t, \tau\}$ uniquely identifies a moment τ within quarter t.

2.2 Representative Household

Households obtain positive utility from the consumption of a final good and supply labor to the firms producing it. They also derive utility from holding real deposit balances in banks, which captures a reduced-form preference for liquidity.[8] The representative household maximizes the following objective function:

$$\max \quad E_t \sum_{j=0}^{\infty} \beta^j \left[\log\left(X_{t+j}\right) - \left(\frac{\eta}{\eta+1}\right) \int_0^1 N_{t+j,\tau}^{1+1/\eta} \, \mathrm{d}\tau \right] ,$$

[7]The continuum can be interpreted as a smooth approximation to the days that comprise a quarter.

[8]Alternatively, utility from real deposit balances can be interpreted as capturing the usefulness of money in the completion of consumption transactions.

where $N_{t,\tau} = \left[\int_0^1 N_{t,\tau}(\nu)^{1+1/\eta}\,\mathrm{d}\nu\right]^{\frac{\eta}{\eta+1}}$ is the aggregate labor index and $N_{t,\tau}(\nu)$ labor supplied to intermediate industry ν, η is the Frisch labor supply elasticity, and variable X_t is a composite of consumption and bank deposit balances. In particular,

$$X_t = C_t + \sum_{n=1}^{N} \int_0^1 \left(1 - T_t^n \cdot z_{t,\tau}^n\right) \frac{D_{t,\tau}^n}{P_t}\,\mathrm{d}\tau \ ,$$

where $C_t = \int_0^1 C_{t,\tau}\,\mathrm{d}\tau$ is aggregate consumption in t, P_t is the aggregate price index of the economy, $D_{t,\tau}^n$ are one-period nominal deposits at bank n paying a (gross) return $R_{t,\tau}^{D,n}$, $(1-T_t^n)$ is the average utility of deposits at bank n, and $z_{t,\tau}^n$ is an exogenous shock to those deposit preferences. We model $z_{t,\tau}^n$ as a Weibull-distributed shock with mean one and shape parameter κ controlling its volatility, and assume the shocks to be i.i.d. across banks n, quarters t, and time continuum τ. In addition to bank deposits, we assume that households also have access to a one-period bond with return R_t^B, which is in zero net supply in equilibrium.

The first-order equilibrium condition for deposit rates is:

$$R_{t,\tau}^{D,n} = R_t^B \cdot T_t^n \cdot z_{t,\tau}^n \ , \qquad \forall n \ . \tag{1}$$

Movements in the return of bonds R_t^B have a proportional impact on the deposit rates paid by all banks, while shocks to individual bank preferences $z_{t,\tau}^n$ alter the relative costs of attracting deposits and lead to a reallocation of deposits across banks at each moment τ.

2.3 Firms

A mass-one of differentiated intermediate goods indexed by ν is in monopolistic competition with the following production function employing capital and labor:

$$Y_{t,\tau}(\nu) = \left(\frac{K_t(\nu)}{\alpha}\right)^{\alpha} \left(\frac{exp(u_t^A)\cdot N_{t,\tau}(\nu)}{1-\alpha}\right)^{1-\alpha} \ ,$$

where u_t^A is an exogenous technology process. Intermediate producers have sticky prices à la Calvo (1983) and reset prices at the beginning of the quarter with probability $1-\theta$. A representative, perfectly competitive firm combines intermediates into a final good Y_t via a CES aggregator with $\epsilon > 1$ elasticity of substitution across varieties.

We assume that firms must employ a constant level of capital throughout the quarter, $K_{t,\tau}(\nu) = K_t(\nu),\ \forall\tau$, so all production adjustments within the quarter occur through the labor margin. Aggregate capital is a CES composite of N distinct types of capital:

$$K_t(\nu) = \left[\sum_{n=1}^{N} (a_t^n)^{1/\sigma} K_t^n(\nu)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \ ,$$

where $\sigma > 1$ is the elasticity of substitution between types, and a_t^n is a demand shock. Capital investment requires one unit of the final good, and without loss of generality to the qualitative results of the paper, we assume full depreciation and instant build-up of capital from investment. Firms finance their investment with credit from N distinct banks,[9] each specialized in the provision of one-period loans $L^n_{t,\tau}(\nu)$ at gross interest rate $R^{F,n}_t$ for the purchase of a distinct type of capital. Solving the firm's optimization problem and adding loan demands across firms, we obtain an expression for bank n's aggregate demand as:

$$L^n_t = a^n_t \left(\frac{R^{F,n}_t}{R^F_t} \right)^{-\sigma} L_t \ , \tag{2}$$

where L_t and R^F_t are aggregate loan and interest rate indices, respectively.[10]

2.4 Banking Sector

Each bank performs three activities: they obtain deposits from the representative household, provide credit to firms, and trade funds with each other in the interbank market. For expositional purposes, we assume that each bank is composed of two divisions, each one responsible for a different set of tasks. The Loan Division provides credit to firms and secures the necessary funding through internal funds or interbank loans. The Deposit Division procures deposits from the representative household and distributes them to the Loan Divisions.

2.4.1 Loan Division

Loan Division n is subject to the following constraints limiting the creation of firm loans:

$$L^n_{t,\tau} \leq M^n_{t,\tau} \ , \quad M^n_{t,\tau} \geq 0 \ , \tag{3}$$

where $M^n_{t,\tau}$ is the amount of internal and/or interbank funding available at time τ. The first constraint restricts banks' credit provision by the amount of available funds and holds with equality in equilibrium. Bank funds are perfect substitutes, and Loan Divisions obtain them as one-period interbank loans (or internal transfer) from the bank that offers the lowest rate. Formally,

$$\begin{aligned} &M^n_{t,\tau} = M^{in}_{t,\tau}, \quad i_{t,\tau}(n) = \arg_j \min \left\{ R^{I,jn}_{t,\tau} \right\} \ , \\ &R^{I,n}_{t,\tau} = R^{I,in}_{t,\tau} \ , \end{aligned} \tag{4}$$

where $M^{in}_{t,\tau}$ are the interbank funds lent by Deposit Division i to Loan Division n and $R^{I,in}_{t,\tau}$ is the gross interbank rate at which bank i is willing to lend to bank n. Banks know their individual firm loan demands given by equation (2) and act as monopolistic competitors, taking the aggregate index R^F_t as given. Banks and firms meet at the beginning of the quarter and agree on an inter-

[9] We do not consider self-financed firm investment, but such distinction would not affect the qualitative results of the model.

[10] Note that from these expressions we can alternatively interpret σ as the elasticity of substitution between loans and a^n_t as a loan demand shock.

est rate that will prevail throughout the period.[11] This results in a constant firm loan demand throughout the quarter, which banks have to finance while experiencing a varying capacity to attract funds due to depositor preference shocks $z^n_{t,\tau}$, forcing them to borrow from the interbank market or, in the absence of trading opportunities, reduce loan demand ex-ante by charging higher interest rates to firms. However, interbank rates are renegotiated at each instant τ and reflect the shifting capacity to provide funds by the emitting bank. Solving the maximization problem of the Loan Division, we obtain the optimal interest rate on firm loans as a constant mark-up over the average cost of funds:

$$R^F_t = \left(\frac{\sigma}{\sigma-1}\right) R^I_t \ , \qquad R^I_t = \left[\sum_{n=1}^{N} a^n_t \left(R^{I,n}_t\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \ ,$$

where $R^{I,n}_t \equiv \int_0^1 R^{I,n}_{t,\tau} \, \mathrm{d}\tau$ is the average interbank rate paid by bank n in quarter t.

2.4.2 Deposit Division

The Deposit Division obtains deposits from the household and converts them into internal funding or interbank loans. The amount of funds that n can provide is given by

$$M^{nn}_{t,\tau} + \sum_{i \neq n} d^{ni}_t \cdot M^{ni}_{t,\tau} = D^n_{t,\tau} \ ,$$
$$\text{subject to:} \quad M^{ni}_{t,\tau} \geq 0 \ , \quad D^n_{t,\tau} \geq 0 \ , \quad \forall n, \, i \ ,$$

where $d^{ni}_t \geq 1$ are transaction costs incurred for the transfer of funds from Deposit Division n to Loan Division i. We interpret these costs as capturing screening, enforcement, or other costs related to an interbank transaction.[12] We implicitly normalize to one the transaction costs between divisions of the same bank, $d^{nn}_t = 1, \ \forall n$. The markets for interbank loans and deposits are perfectly competitive, and banks act as price takers.[13] Solving the optimization problem of the Deposit Division and using equation (1), we obtain an expression for the interbank loan rate offered by bank n to bank i,

$$R^{I,ni}_{t,\tau} = R^B_t \cdot d^{ni}_t \cdot T^n_t \cdot z^n_{t,\tau} \ . \tag{5}$$

[11]An alternative assumption with equivalent results would be that firm interest rates are sticky *within* the continuum and can only be reset at the beginning of each quarter. Sørensen and Werner (2006) provide supporting empirical evidence for the stickiness of retail interest rates.

[12]For example, uncertainty surrounding the value of mortgage-backed securities (and related assets) following the 2007 financial crises can be interpreted through the lens of the model as an increase in the d^{ni}_t costs of collateral screening.

[13]Alternatively, modeling the interbank market under Bertrand competition still brings a tractable solution to the problem but complicates the welfare analysis considerably.

Using the previous equation, we obtain an expression for bank n's average interbank borrowing credit spread as

$$E_t\left[\min_{i\in\{1,\ldots,N\}}\left\{R^{I,in}_{t,\tau}\right\}\right]/R^B_t=\left[\sum_{i=1}^{N}\left(d^{in}_t\cdot T^i_t\right)^{-\kappa}\right]^{-1/\kappa}\equiv\Phi^n_t\ ,$$

where $d^{in}_t\cdot T^i_t$ is the average spread at which bank i is willing to lend funds to bank n, and which is determined by the bilateral transaction costs d^{in}_t as well as the efficiency T^i_t with which bank i attracts funds from its own depositors.

2.5 Central Bank

The central bank can affect the risk-free rate of the economy through conventional open market operations as well as provide direct credit to banks in the system in its role as lender-of-last-resort (LoLR). We describe both types of intervention in this section.

2.5.1 Lending Facility

The central bank provides direct credit to banks via its lending facilities[14] and other LoLR interventions. We assign subindex zero to the central bank and model it as an additional bank within the system with some unique characteristics. Namely, the central bank does not raise deposits from households, has the capacity to freely create money, and therefore can arbitrarily set the interest rate at which it offers funds to the system banks. Consistent with most historical discount window policies, we model the lending rate charged by the central bank as a penalty rate over the average cost-of-funds at which each bank n is able to borrow from the rest of its funding sources. Formally, this relationship is represented as

$$R^{I,0n}_{t,\tau}=\chi^n_{t,\tau}\cdot\Phi^n_t\cdot R^B_t\ , \tag{6}$$

where $\chi^n_{t,\tau}$ is the penalty rate.[15] We study different lending policies by assigning a flexible functional form to the penalty rate:

$$\chi^n_{t,\tau}=e^{\varpi_1}\cdot\underbrace{\left(\frac{\Phi^n_t}{\Phi^n}\right)^{-\varpi_2}\cdot z^0_{t,\tau}}_{\text{variable component}}\ ,$$

where ϖ_1 is a parameter that controls the steady state size of the penalty, and ϖ_2 its response to steady state deviations of the bank's borrowing spread. $z^0_{t,\tau}$ is a policy shock, which we introduce for analytical convenience and assume to be distributed Weibull with mean one and shape parameter κ. For analytical tractability, we assume that any profits made by the central bank on its lending operations are returned to the household via lump-sum transfer.

[14]Examples include the ECB's marginal lending facility or the Fed's discount window.

[15]More broadly, penalty $\chi^n_{t,\tau}$ can also be thought of as capturing other costs of accessing central bank credit, such as discount window stigma.

2.5.2 Taylor Rule

The central bank also determines the nominal risk-free rate R_t^B of the economy through conventional open market operations. We assume that it follows a Taylor rule of the form

$$R_t^B = R^B \cdot \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\gamma_y} \cdot exp\left(u_t^R\right) \ , \tag{7}$$

where $\Pi_t \equiv P_t/P_{t-1}$ stands for gross inflation, Y_t^n is output under flexible prices and u_t^R is an exogenous monetary policy shock.

2.6 Government

Section 3 derives an approximation to welfare under the assumption that the government eliminates real economic distortions via subsidies funded through lump-sum taxation of the representative household.[16,17] The first set of subsidies targets under-production by intermediate good producers due to monopolistic pricing. Similarly, a second set of subsidies corrects the distortions imposed by banks' monopolistic competition in the provision of firm loans, which would otherwise result in the under-accumulation of capital. Finally, a third set of subsidies to deposits corrects steady state distortions to the household's savings rate induced by the central bank's provision of credit via the lending facility. This distortion is specific to our setting and follows from the central bank creation of funds, which results in a lower interest rate paid to depositors as banks substitute household deposits in favor of lending facility borrowing.[18]

2.7 Shock Processes

We define the functional form of transaction costs shocks as

$$d_t^{ni} = \begin{cases} \left(d^{ni}\right)^{\varrho} \cdot exp(u_t^{I,ni}) & \text{, if } i \neq n \ , \\ 1 & \text{, otherwise.} \end{cases} \quad , \qquad u_t^{I,ni} = \rho_I \cdot u_{t-1}^{I,ni} + \varepsilon_t^{I,ni} \ , \quad \forall n, i \ , \tag{8}$$

where $\varepsilon_t^{I,ni}$ are mean-zero, exogenous (but potentially correlated) stochastic shocks. Parameter ϱ will allow us to modify the size of transaction costs once we look at banking system integration counterfactuals in section 7. The shock processes for loan demand a_t^i and depositor preferences T_t^i follow a similar autoregressive structure as in equation 8 with stochastic shocks $\varepsilon_t^{a,i}$ and $\varepsilon_t^{T,i}$.

[16] This assumption greatly simplifies the analytical tractability of the welfare expression and is common practice in the macroeconomics literature, see for example Woodford (2003).

[17] Subsidies are not crucial to the derivation of the equilibrium conditions of the model, and for simplicity, we report the main analytical expressions of this paper assuming no government subsidies, unless otherwise explicitly stated. Appendix 1 provides a detailed derivation of the model conditions, including the subsidy schemes.

[18] The smoothness induced by the extreme value liquidity shocks z_t^n ensures that banks borrow a non-zero amount from the central bank's lending facility in steady state, which prevents this distortion from disappearing. Nonetheless, steady state borrowing from the lending facility is low for reasonable calibrations, and therefore we choose to offset this distortion via subsidy in favor of analytical tractability of section 3's welfare expressions. From an empirical standpoint, we can justify the assumption on deposit subsidies by the fact that there is no evidence of LoLR interventions depressing the steady-state interest rate paid on bank deposits.

2.8 Banking Sector Aggregation & Equilibrium Conditions

Plugging equations (5) and (6) into (4), we obtain an expression for the average interbank credit spread paid by bank n as

$$\widetilde{R}_t^{I,n} \equiv \frac{R_t^{I,n}}{R_t^B} = \Phi_t^n \cdot \left(1 - \xi_t^{0n}\right)^{1/\kappa} \ , \tag{9}$$

where ξ_t^{0n} is the share of total funding obtained from the central bank's lending facility.[19] We can express the share of non-central bank funding and total borrowing volume that bank i obtains from bank n as

$$\lambda_t^{ni} = \left(\frac{d_t^{ni} T_t^n}{\Phi_t^i}\right)^{-\kappa} \ , \qquad \forall n \in \{1, \ldots, N\} \ , \tag{10}$$

$$M_t^{ni} = \left(1 - \xi_t^{0i}\right) \cdot \lambda_t^{ni} \cdot L_t^i \ , \tag{11}$$

Intuitively, $d_t^{ni} \cdot T_t^n$ is the average spread above the risk-free rate at which bank n funds are *offered* to bank i over the quarter, while Φ^i is the average credit spread at which i *effectively* borrows. A larger difference between the spreads implies that bank n is less likely to be the least-cost supplier of funds to bank i at any given moment τ, and hence the quarterly transaction volume flowing from n to i is smaller, both in absolute and relative terms. Also, note that our setting assumes perfect substitution between interbank funding sources (from the borrower's perspective), but generates a quarterly downward-sloping demand for interbank loans with constant elasticity of substitution.[20,21] Finally, notice that the variance of depositor preferences κ can be alternatively interpreted as the interbank supply elasticity in (10).[22]

[19] An analytical expression for this term is given by $\xi_t^{0n} = \left[1 + e^{\kappa \varpi_1} \cdot \left(\frac{\Phi_t^n}{\Phi^n}\right)^{-\kappa \varpi_2}\right]^{-1}$.

[20] This result follows from the volatility of depositor preferences $\left\{z_{t,\tau}^j\right\}_{j=1}^N$, which ensure that all banks eventually experience episodes of high deposit influx and momentarily become the least-cost suppliers of their connections.

[21] The interbank loan demand of our model could be alternatively obtained as —and is isomorphic to— a love-for(-interbank)-variety demand. Compared to that alternative formulation, our setting provides a plausible microfoundation for modeling liquidity fluctuations in financial markets.

[22] The intuitive link between both interpretations of κ comes from the fact that a low variance of depositor preferences (high κ values) makes it less likely for an excess influx of deposits to compensate the differences between supply $d_t^{ni} \cdot T_t^n$ and borrowing Φ^i spreads.

2.9 Steady-state Relationships

In steady state, the aggregate share of funds originating from banks' own sources (*own share*) and the aggregate central bank share are defined, respectively, as

$$\lambda_t^{Own} = \left[\sum_{n=1}^{N} s_t^n \cdot \left(\lambda_t^{nn}\right)^{\frac{\sigma-1}{\kappa}}\right]^{\frac{\kappa}{\sigma-1}} , \qquad \text{where} \quad s_t^n = a_t^n \cdot \left(\frac{\widetilde{R}_t^{I,n}}{\widetilde{R}_t^I}\right)^{1-\sigma} .$$
$$\xi_t^0 = \sum_{n=1}^{N} s_t^n \cdot \xi_t^{0n} ,$$

An expression for the steady state credit spread is given by

$$\widetilde{R}^I = \left(1-\xi^0\right)^{1+1/\kappa} \cdot \left(\lambda^{Own}\right)^{1/\kappa} \cdot \left[\sum_{n=1}^{N} a^n \cdot \left(T^n\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} .$$

A higher central bank funding share reduces the credit spread, as lender-of-last-resort (LoLR) interventions set an upper bound on the funding costs of banks and narrow the gap between the interbank and the risk-free rate. The own share is inversely related to the credit spread, capturing the fact that banks gain access to cheaper funding sources by participating in the interbank market. Figures 2 and 3 provide suggestive evidence for this relationship, with the aggregate funding from oneself and the Euribor credit spread converging to permanently higher levels following the 2007 financial crisis. The last term captures the preference of households for holding money as bank deposits vis-à-vis risk-free bonds and implies that stronger preferences (lower T^n's) reduce the return on deposits that households are willing to accept, narrowing the interest rate gap between the two.

2.10 Log-Linearized System

This section presents the dynamic solution of the model under a first-order approximation. We use lowercase letters to denote the logarithm of a variable, and hats refer to deviations from the steady state. We discuss the key equilibrium equations here and relegate the derivations of the approximation to Appendix 1.

The system is comprised by a New-Keynesian Phillips Curve and a Dynamic IS Equation, respectively:

$$\hat{\pi}_t = \Omega\,\hat{\tilde{y}}_t + \beta E_t\left[\hat{\pi}_{t+1}\right] ,$$
$$\hat{\tilde{y}}_t = -\left[1+\alpha\left(\frac{\eta}{\eta+1}\right)\right] \cdot \left[\hat{r}_t^B - E_t\left[\hat{\pi}_{t+1}\right] - \hat{\iota}_t^n\right] + E_t\left[\hat{\tilde{y}}_{t+1}\right] ,$$

where $\tilde{y}_t \equiv y_t - y_t^n$ is the output gap, Ω is a constant and ι_t^n is the natural interest rate under flexible prices, which in turn is a function of the aggregate interbank rate, $\tilde{r}_t^I$, and the central bank's interbank funding share, $\widehat{\log\left(\xi_t^0\right)}$.[23] The term $\left[1+\alpha\left(\frac{\eta}{\eta+1}\right)\right]>1$ in the Dynamic IS Equation captures the sensitivity of output gap to deviations of the real interest rate from its natural counterpart.[24] The expressions for the interbank rate and the central bank funding share are given by

$$\hat{\tilde{r}}_t^I = \rho_I \cdot \hat{\tilde{r}}_{t-1}^I + (1-\varpi_2\xi^0)\left[\varepsilon_t^T+\varepsilon_t^I\right] - \frac{\varepsilon_t^a}{\sigma-1} \ ,$$
$$\widehat{\log\left(\xi_t^0\right)} = \rho_I \cdot \widehat{\log\left(\xi_{t-1}^0\right)} + \kappa\varpi_2(1-\xi^0)\cdot\left[\varepsilon_t^T+\varepsilon_t^I\right] \ ,$$

where $\varepsilon_t^T,\ \varepsilon_t^I$ and ε_t^a are average combinations of the individual-bank shocks to depositor preferences, transaction costs, and firm loan demand, respectively. The underlying structure of the banking system determines the volatility of these shocks, as the combination of the different individual-bank shocks depends on the steady-state interbank bilateral trade shares λ^{ni} and the share of the firm loan market s^i controlled by each bank. Note also that terms related to the central bank's lending facility $\{\varpi_2,\ \xi^0\}$ enter the above equations multiplying the aggregate shocks. This points towards the capacity of discount window policies to dampen the transmission of banking shocks, as we should see later.

3 Welfare

3.1 Gains from Trade

In this section, we derive an analytical formula for the steady-state gains from trade in interbank markets and extend this formula for dynamic gains based on a second-order approximation of our model. We define autarky (AU) as the counterfactual economy without interbank markets, in which banks exclusively fund their operations through deposits and credit from the central bank's lending facility.[25] We define the gains from interbank trade as the change in the expected household's utility relative to autarky, formally

$$\mathbb{J} \equiv E\left[\frac{U_t - U_t^{AU}}{U_X X}\right],$$

where $U_X \equiv \mathrm{d}U/\mathrm{d}X$ normalizes welfare as a fraction of steady-state consumption of X. Gains from trade represent a conceptually distinct notion of welfare from the typical macro definition

[23]The expression for the natural interest rate is given by

$$\iota_t^n \equiv \left[(1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right)\cdot\tilde{r}_t^I - (1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\eta+1}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\cdot\log\left(\xi_t^0\right) - (1-\rho_A)\cdot u_t^A\right] \ .$$

[24]This term equals one in standard New Keynesian models, and the additional sensitivity in our setting comes from the inclusion of non-separable utility for deposits on the household's utility function. See Fisher (2015) for a discussion on how non-separable preferences for assets in the utility due to liquidity and/or safety motives modify the dynamic IS equation of the standard New Keynesian model.

[25]We achieve this equilibrium by letting transaction costs approach infinity ($\varrho\to\infty$) in equation (8).

of welfare as business cyclical costs of volatility. The increase in interbank participation (from initial autarky) not only influences the economy's volatility but also shifts its steady state, potentially leading to substantial first-order welfare gains. To further understand the factors driving gains from trade in our model, we conduct a second-order approximation of the above expression around the steady state (Proof: see Appendix 2)

$$\mathbb{J} = \mathbb{J}^{ss} + \frac{1}{2}\left[\sigma_T^2 \cdot \mathfrak{J}^T + \sigma_a^2 \cdot \mathfrak{J}^a + \sigma_I^2 \cdot \mathfrak{J}^I\right] + \text{h.o.t.}\,, \tag{12}$$

where h.o.t. represents the approximation error accounted for by higher-order terms, and $\mathbb{J}^{ss} \equiv \frac{U-U^{AU}}{UXX}$ are the static gains from trade defined as the steady-state difference in utility relative to autarky.[26] Parameters σ_T^2, σ_a^2, and σ_I^2 refer to the volatility of depositor preferences, firm loan demand, and transaction cost shocks, respectively.[27] Multipliers $\mathfrak{J}^m,\ m \in \{T, a, I\}$ measure the change in the welfare costs imposed by the volatility of these shocks due to market integration, with positive (negative) values indicating a reduction (increase) in the cyclical costs of these shocks.

We continue our investigation by separately examining the components of (12). The expression for the static gains from trade, $\mathbb{J}^{ss}$, becomes a function of the underlying structural parameters as[28]

$$\mathbb{J}^{ss} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{\kappa} \cdot \log\left(\lambda^{Own}\right)\ \geq\ 0\,. \tag{13}$$

Interbank integration, implicitly captured by a low share of internal funding λ^{Own}, yields monotonically increasing welfare gains via the efficient allocation of funds across the banking network. These gains are further amplified when the substitution of funding across interbank connections is difficult (low supply elasticity κ) and when capital financing becomes more important for production (high capital share α).[29] A significant advantage of this expression for policy practitioners is that λ^{Own} becomes a sufficient statistic of the gains from trade that is readily observable in the data.[30]

We now introduce the following metrics for assessing banking sector concentration:

$$H_t^F = \sum_{n=1}^{N} \left(s_t^n\right)^2\ , \quad \text{and} \quad H_t^{I,n} = \sum_{j=1}^{N} \left(\lambda_t^{jn}\right)^2\ , \forall n\ ,$$

[26] $\mathbb{J}^{ss}$ serves as the point of approximation for equation (12). This is consistent with the practice commonly employed by the macroeconomics literature of approximating welfare around the efficient steady state.

[27] We obtain this expression by assuming equal volatility of shocks across banks. See Online Appendix Appendix 2 for a description of the assumptions involved in equation (12) approximation.

[28] Note the similarity of this expression with the gains from trade commonly found in the international trade literature, e.g., Arkolakis, Costinot, and Rodríguez-Clare (2012).

[29] The $1-\alpha$ term in equation (13) follows from an input-output multiplier effect between loan supply and output, resulting in the creation of additional capital investment.

[30] Equation (13) provides an *ex-ante* measure of welfare, in the sense that it does not require knowledge about the underlying structure of the model nor information on the counterfactual autarky scenario.

where H_t^F denotes the Herfindahl-Hirschman index (HHI) of bank concentration in the provision of loans to firms, while $H_t^{I,n}$ represents the HHI for the concentration of funding sources for bank n. These indices provide a quantitative measure of concentration for both the final services rendered by the banking sector (i.e., loans to firms) and its inputs (i.e., bank funding). This information is valuable for understanding how alterations in the network structure influence welfare. For instance, the multiplier related to the volatility of transaction costs, denoted as $\mathfrak{J}^I$, is proportional to the following expression:[31]

$$\mathfrak{J}^I \propto \underbrace{\sum_{n=1}^{N} s^n \cdot [1 - \lambda^{nn}]}_{\text{Diversification}} - \underbrace{\left[\Theta_0 + \Theta_1 \cdot H^F\right] \cdot \sum_{n=1}^{N} \omega^n \cdot \left[H^{I,n} - (\lambda^{nn})^2\right]}_{\text{Exposure}} ,$$

where Θ_0 and Θ_1 are constants, and ω^n are weights that sum to one. A more detailed derivation can be found in Appendix 3. The *Diversification* component captures the advantages arising from reduced funding volatility, as banks participating in the interbank market gain access to alternative funding sources. Conversely, participation in the interbank market exposes banks (and the firms with sticky prices that borrow from them) to unanticipated interbank transaction cost shocks (and consequently, funding cost fluctuations) that are absent in autarky.[32] These additional costs are represented by the *Exposure* component and become more pronounced when external interbank funding sources exhibit higher concentration, as indicated by a greater $H^{I,n} - (\lambda^{nn})^2$ term, or when the firm's loan market is dominated by a few banks, as captured by the H^F index.

The multipliers $\mathfrak{J}^T$ and $\mathfrak{J}^a$ correspond to the variations in deposit and firm loan concentration among banks, respectively. An expansion in the interbank market that results in an increased (decreased) market share for larger banks also amplifies (reduces) the overall volatility within the economy. The rationale for this outcome is that, given a sufficiently large number of banks ($N \to \infty$), the influences of depositor preference shocks and firm loan demand disturbances cancel out in the aggregate, provided that the distribution of deposits and loan supply is uniformly dispersed among banks. When this condition is not satisfied due to banking sector concentration, individual shocks generate aggregate effects.[33] Consequently, whether an expansion of the interbank market results in second-order welfare gains or losses is ultimately an empirical question contingent upon the specific integration patterns of financial markets.

[31]This expression is derived under the simplifying assumption of uncorrelated transaction cost shocks. Refer to Appendix 2 for a more comprehensive formulation that accounts for a general correlation structure.

[32]Recall that autarky transaction costs with oneself are constant, as we normalized $d_t^{nn} = 1,\ \forall n,\ t$ in (8).

[33]Huber (2018) presents empirical evidence supporting this result by demonstrating that credit shocks to *Commerzbank* (a prominent German MFI) had significant impacts on the German economy during the 2008/2009 financial crisis.

3.2 Gains from the Lender-of-Last-Resort (LoLR)

Similar to gains from trade, we quantify the welfare of different LoLR policies by comparing them to the counterfactual equilibrium without LoLR credit provision, formally

$$\mathbb{G} \equiv E\left[\frac{U_t - U_t^{no-LoLR}}{U_X X}\right] .$$

The steady-state share of credit supplied by the central bank is

$$\xi^0 = \frac{1}{1 + e^{\kappa\varpi_1}} ,$$

where $\varpi_1 \to +\infty$ corresponds to the counterfactual scenario. The interbank volatility that the central bank aims to manage arises from two distinct sources in our model, namely: intra-quarter volatility, related to the reshuffling of deposits across banks, which results in temporary funding shortfalls, and inter-quarter volatility, originating from persistent variations in transaction costs. Parameter ϖ_1 affects the former source of volatility by setting an upper limit to the funding costs of banks, leading to more stable and (on average) cheaper funding *within* any given quarter. Conversely, parameter ϖ_2 controls the volatility and cyclicality of LoLR credit *across* quarters.

A second-order approximation to the gains from LoLR can be found in Appendix 3. When $\varpi_2 = 0$, the central bank imposes a fixed penalty over the average cost of funds, $\widetilde{R}_t^{0n} = e^{\varpi_1} \cdot \Phi_t^n$, and the expression simplifies to

$$\mathbb{G} = -\left(\frac{\alpha}{1-\alpha}\right) \cdot \left(1 + \frac{1}{\kappa}\right) \cdot \log\left(1 - \xi^0\right) \;\geq\; 0 \,. \tag{14}$$

Observe that the gains from LoLR are monotonically increasing in steady-state central bank participation, ξ^0, which summarizes the benefits of lower intra-quarter volatility and funding costs. The general case with $\varpi_2 \neq 0$ adds additional potential gains to the previous expression by also reducing the welfare costs of inter-quarter interbank volatility, as we will see in the quantification exercises of section 7.

4 Data

In this section, we describe how we combine several proprietary and confidential datasets provided by the Deutsche Bundesbank, the German central bank (within the system of Eurozone central banks) and supervisory entity for the German financial market. The resulting database contains detailed, quarterly information on the balance sheets of monetary financial institutions (MFI), borrowing and lending connections with other German MFIs and other information such as the type of banking group for the universe of German MFIs covering 2004-2016. Our dataset contains 1552 distinct MFIs.[34] [35]

For the construction of this database, we start with the MFI Masterdata (MaMFI[36]), which includes information such as MFI type and headquarter location on a monthly basis and allows us to account for mergers and acquisitions in all other datasets.[37] Next, we add the Monthly Balance Sheet Statistics (BISTA[38]) that covers broad balance sheet positions of the universe of German domestic MFIs at the end of each month. To better account for each MFIs' business model, we complement the broad loan categories in the BISTA with a more detailed breakdown of loans by sectors, borrower type and maturities from the Quarterly Borrowers' Statistics (VJKRE[39]).

Finally, the Credit Register of Loans (Millionenkreditregister) provides MFI-level supervisory information on all loans that exceeded 1 million Euro (1.5 million Euro before 2014) in each quarter. The dataset contains various loan types, but most importantly, it covers the vast majority of loans at all maturities between domestic MFIs.[40] Due to this censoring, we scale individual positions between two MFIs such that the total MFI lending in this dataset is consistent with domestic MFI loans in the BISTA data. Bilateral loan amounts allow us to capture the interbank market network in its full granularity over many years throughout our empirical and model-based analysis.

We complement our main database with interest rate data for a sample of 200 to 240 MFIs representing 65% to 70% of total lending activities in the banking sector.[41] The Monthly Interest Rate Statistics (ZISTA[42]) reports average interest rates on loans and deposits vis-à-vis firms and households and their respective volumes. For most of our analysis, we use the average interest rate on outstanding loans for each MFI that we calculate as the average interest rate across all maturities and borrower types weighted by their respective loan volumes.[43]

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[34] Monetary and financial institutions are defined by the European Central Bank as "financial institutions whose business is to receive deposits and/or close substitutes for deposits from entities other than MFIs and, for their own account (at least in economic terms), to grant credits and/or make investments in securities", https://www.ecb.europa.eu/stats/financial_corporations/list_of_financial_institutions/html/index.en.html. However, for the remainder of the paper we will us the terms "MFI" and "bank" interchangeably.

[35] There are 9 different banking groups in the Deutsche Bundesbank statistical definition. The largest among them are credit banks, state banks, savings banks, mortage banks and cooperative banks.

[36] See Stahl (2018) for a description of the MaMFI dataset.

[37] In order to avoid sudden discontinuities in the balance sheet size and its subcategories, we treat MFIs before and after a merger or acquisition as a single entity and add up the relevant categories for the MFIs participating in the M&A.

[38] See Beier, Krueger, and Schaefer (2017) for a description of the BISTA dataset.

[39] See Beier, Krueger, and Schaefer (2018) for a description of the VJKRE dataset.

[40] We find that a comparison of liabilities and assets with MFIs in the balance sheet data and aggregated loans in the credit registry line up very tightly. This suggest that the reporting threshold of 1 million Euro (1.5 million Euro before 2014) is a not serious concern for lending between MFIs.

[41] The sample is designed to be representative and, at the same time, capture a large share of the financial sector. The first stratification criterion is a combination of state and banking group in order to capture regional and institutional heterogeneity. Within each of the strata, the largest banks in terms of lending were selected. Throughout our analysis we tried to address this selection bias whenever possible.

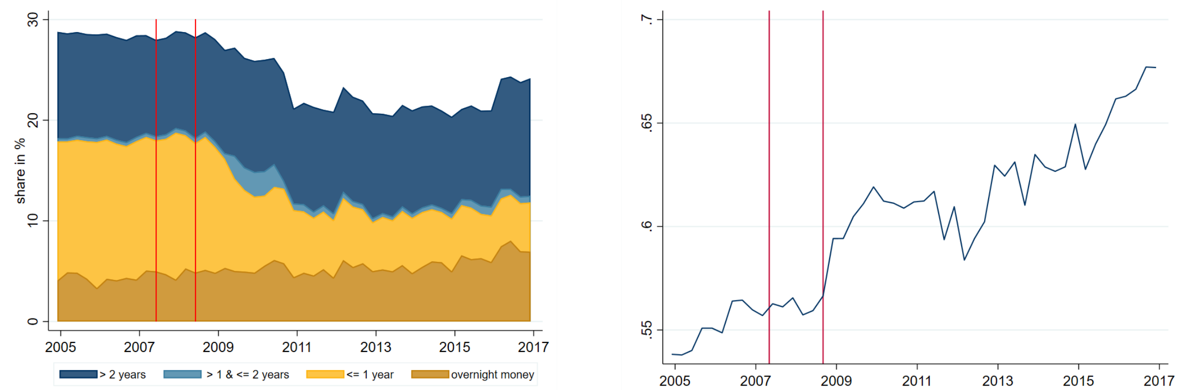
[42] See Beier and Bade (2017) for a description of the ZISTA dataset.

[43] Another adjustment to our data comes from the 2010 German Accounting Modernization Law (see Bundesbank (2010) for a description of the Accounting Modernization Law), that, among other changes for firms, caused a break in banks' balance sheet sizes. Generally, the most prominent change was the introduction of a fair value of the trading portfolio, partly adapting to the International Financial Reporting Standards (IFRS). While this change affects only

Lastly, we measure the exposure of each domestic MFI to the US financial market before the Great Recession using the assets and liabilities of German banks vis-à-vis US residents (including banks) available in the External Position of MFIs (AUSTA[44]). This dataset contains the gross foreign positions by partner country of the 80 largest German banks and their foreign branches on a monthly basis, covering 90% of the value all foreign positions involving a German MFI.

5 The German Interbank Market

In this section, we present several facts about the structure of the German interbank market that inform the quantification of our welfare formula introduced in section 3. As of December 2016, banks, with a combined balance sheet of 7.8 trillion Euros (approximately 250% of GDP), serve as the central players in the German financial sector. This is in contrast to investment funds (1.9 trillion in assets), insurers (2.2 trillion in assets), and other financial services providers.



(a) Aggregate Interbank Liabilities

(b) Aggregate Own Share

Figure 2: (a) Share of interbank liabilities in total liabilities by maturity. (b) Aggregate Own Share defined as $1 - \frac{\text{Interbank Liabilities}}{\text{Assets} - \text{Interbank Assets}}$. Red lines indicate $2007Q2$ and $2008Q2$ as the starting period for the financial crisis. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA, 2004m12 - 2018m12, own calculations.

Panel (a) of Figure 2 reveals that the combined liabilities in the interbank market as a share of the aggregate balance sheet remained stable at approximately 29%, with a 6 percentage point drop following the 2007/2008 crisis. Throughout the sample period, over two-thirds of bank-to-bank liabilities had maturities longer than overnight, challenging the common perception that interbank markets primarily serve for very short-term funding. Panel (b) displays the share of loans to firms and consumers that banks can fund with their own resources (deposits or equity), as opposed to borrowing on the interbank market.[45] Following the crisis, banks progressively

larger banks with a trading book, it was left to their discretion at what point in the course of 2010 they applied the new rules in monthly balance sheet statistics (BISTA). We mitigate this circumstance by deducting the derivative exposures of the trading book from total assets.

[44]See Gomolka, Munzert, and Stahl (2019) for a description of the AUSTA dataset.

[45]The aggregate own share in panel (b) is not the same as one minus the interbank liability share. The reason is that

began relying more on their own funding sources, with no apparent sign of reverting to previous levels. Our formula in section 3 suggests that this drop in the own share leads to persistent welfare losses, which we will investigate further in section 7.

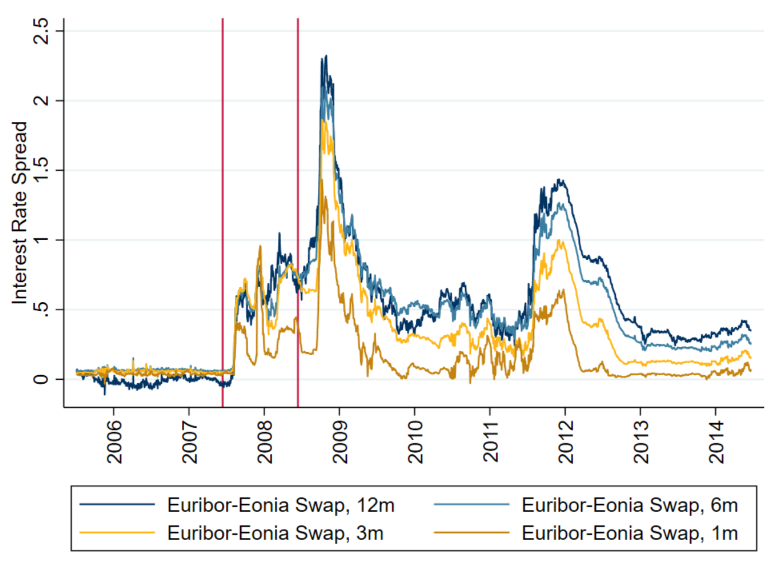


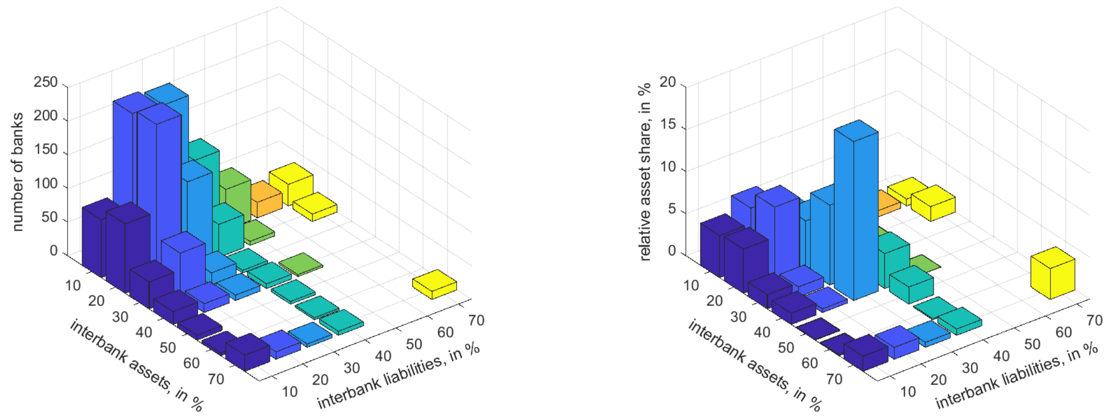
Figure 3: Interbank credit spread at different maturities, computed as the difference between the Euribor rate and the EONIA swap index. The Euribor is an average of the unsecured interbank rate at which Eurozone banks are willing to lend funds to each other. The EONIA Swap is a financial instrument commonly used to hedge against overnight moves of the unsecured interbank rate. Red lines show $2007Q2$ and $2008Q2$ as the starting period for the financial crisis. Source: Deutsche Bundebank

Figure 3 displays the evolution of the interbank credit spread at various maturities, calculated as the difference between Euribor and Eonia Swap rate. Two significant spikes are observed following the 2007 financial crisis and the European debt crisis. After 2013, the spread stabilizes, but it does not return to pre-crisis levels, particularly at longer maturities. Through the lens of our model, we interpret the fluctuations in credit spreads as (correlated) shocks to transaction costs that led to the drop in interbank liabilities, as described earlier.

We now examine the internal structure of the German interbank market. A prominent feature of the market is its high degree of concentration. The three largest banks control approximately 30% of the market, and the 200 largest banks (out of more than 1500) control around 80%.[46] Figure 4a categorizes all MFIs into bins based on their share of interbank assets and liabilities in their balance sheets and reports the number of entities within each. In contrast, Figure 4b displays the share of total assets contained in each bin. We first observe that a substantial fraction of banks hold large gross positions as both borrowers and lenders simultaneously, which aligns with our model mechanism suggesting that the interbank market serves to cover temporary liquidity shortfalls. However, as noted by Craig and Ma (2022), numerous banks adopt a net

the denominator in panel (b) captures all assets excluding interbank assets, a measure of loans to the real economy.

[46] Figure 9 illustrates the cumulative share of total assets held by the n-largest banks as of December 2016.



(a) Bank Count

(b) Asset Share

Figure 4: Share of interbank assets and liabilities on the balance sheet, expressed in percentages. The vertical axes in Figure 4a display the number of banks within each bin. Figure 4b presents the share of total MFI assets represented by banks within the bin. Bins with fewer than three observations are not reported due to confidentiality requirements. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA, December 2016, own calculations.

lender or borrower position in the market, with net positions exceeding 10% being common. Consequently, the interbank market not only facilitates short-term liquidity provision but also enables banks to cover structural funding deficits and allocate structural funding surpluses. To

Table 1: Spearman rank correlation tests

Months	3	6	12	24
Interbank asset share	.960 (.001)	.940 (.001)	.912 (.001)	.853 (.002)
Interbank liability share	.922 (.001)	.887 (.001)	.846 (.002)	.768 (.002)
Interbank net position	.970 (.001)	.955 (.001)	.934 (.001)	.888 (.001)

We construct the table by ranking the interbank market share of each bank and estimating the correlation with the ranking m-Months ahead. The first row contains the correlation of the interbank asset share, the second row displays the correlation of the liability share, and the third row presents the correlation in the interbank balance. All coefficients are statistically significant at a threshold of ¡1%. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA, December 2004 to December 2018, own calculations.

demonstrate that such funding gaps are a stable feature of the market, we provide estimates of the Spearman rank correlation for the share of interbank assets and liabilities, as well as their difference in Table 1. The high correlation at horizons of up to two years indicates that banks' positions are indeed very persistent over time. A market for structural funding is also consistent with the non-trivial share of medium and long-term borrowing observed in Figure 2. The distinction between the two roles of interbank markets is relevant, as a market freeze, such as the one experienced after 2007, can result in stronger effects on the real economy if banks are unable to cover their structural positions, thereby being forced to cut back lending to the non-financial sector.[47] Despite funds being treated as a homogeneous good, our model can accommodate temporal liquidity (large gross positions) and structural borrowing (net positions) in the interbank market and adequately capture the relative importance of each role for the aggregate economy.

A comparison between panels (a) and (b) of Figure 4 also suggests that a few large MFIs play a central role in the market (see bin on the 40% assets, 30% liabilities position). Large banks have access to a diversified pool of funding sources, with close to 150 unique interbank lenders, whereas smaller banks typically have no more than 20 connections.[48] Accounting for the high degree of granularity in our model is key to contagion risk, as shocks to large lenders can create aggregate volatility in the financial sector, as in Huber (2018), and similarly to how large firms drive economic fluctuations in Gabaix (2011).

6 Estimation and Calibration

In this section, we take the model to detailed data on the Germany's banking sector introduced in section 4. First, by examining the propagation of the US financial crisis to German banks through their interbank connections, we underscore the key mechanism in the model, specifically, that a bank's interbank market access is a crucial driver of its lending decisions, interest rates, and funding choices. Using domestic banks' *indirect exposure* to the crisis as a plausibly exogenous funding cost shock, we then estimate loan demand elasticity σ and interbank supply elasticity κ. The former represents the degree to which firms substitute between banks in loan demand. The latter governs the volatility of preference shocks for bank deposits and, consequently, the supply of funds into the interbank market. Lastly, we recover the model-implied "wedges" (i.e., loan demand parameters $\hat{a}_t^n$, deposit supply parameters $\hat{T}_t^n$, and bilateral transaction costs $\hat{d}_t^{ni}$) for each bank and quarter using our data and estimated elasticities from the previous step.

6.1 Measuring Banks' Indirect Exposure to the US Financial Crisis in 2008

Prior to 2007/2008, several German banks were heavily invested in loans (broadly defined) to banks domiciled in the US. With the onset of the US financial crisis, German banks *directly* exposed to US bank assets experienced serious liquidity problems, as highlighted by Huber (2018) in the case of the two largest German banks (Deutsche Bank and Commerzbank), and signifi-

[47] In section 6, we show that net borrowers indirectly exposed to the US crisis indeed experienced larger increases in interest rates, cut lending more, and relied more heavily on own funds compared to net lenders after the crisis.

[48] Figure 10 plots the average number of distinct borrowing connections by deciles over the sample period. Deciles in panel (a) of Figure 10 are based on the number of connections, while those in panel (b) are based on bank balance sheet size. The similarity between the two graphs indicates that central positions in the market are highly correlated with bank size and very persistent. On the asset side, a similar pattern emerges, with large banks acting as lenders to the rest of the system (available upon request).

cantly reduced their lending activity in both the German real economy and the interbank market.

To emphasize the role of interbank market access in transmitting the US financial crisis to the German banking sector and the real economy, we focus our analysis on banks that used to borrow from *directly* exposed banks in the domestic interbank market.[49] Specifically, we construct a measure of each bank's *indirect* exposure to the US financial market prior to the Great Recession according to:

$$Exposure_{t0}^{US,n} = \sum_{i \neq n}^{N} \frac{M_{t0}^{in}}{\sum_{i' \neq n}^{N} M_{t0}^{i'n}} \mathcal{M}_{t0}^{US,i} \,. \tag{15}$$

The first component, $\mathcal{M}_{t0}^{US,i}$, represents the value of assets (in billions of Euros) that bank n's lenders report with US banks in $t0$, as reported in the External Position of MFIs (AUSTA) dataset.[50] We weigh each lender i's direct exposure by bank n's liabilities M^{in} with lender i out of n's total interbank liabilities in the initial period $t0$ and sum over all possible lenders of n. Banks with higher *indirect* exposure either borrow heavily from *directly* exposed lenders or have lenders that are heavily invested in the US banking sector.

For the base period $t0$, we choose 2006Q1, six quarters before the first signs of the financial crisis in 2007Q2, which we set as our event date for the onset of the US financial crisis, and ten quarters prior to the collapse of Lehman Brothers in 2008Q3. Generally, selecting the exact event date for the financial crisis proves to be somewhat ambiguous. Hence, we opt to present a non-parametric event study with 2007Q3 as the single event date but exclude 2007Q3-2008Q2 as the event "period" from the sample in the model estimation below. We conclude the sample in 2011Q4 to avoid confounding the effect of the US financial crisis with the subsequent Euro-crisis. In our sample of 182 unique banks, the mean indirect exposure is 2.3 billion Euros, 950 million Euros at the 25th percentile, and 3.4 billion at the 90th percentile.[51]

Our identification strategy is based on the idea that German banks, which are *directly* exposed to the US financial crisis due to their asset position in US banks, must reduce lending in the domestic interbank market once the crisis intensifies in 2008.[52] With our exposure measure, we capture the fact that other domestic banks, which rely on directly exposed banks for their funding, are *indirectly* exposed and consequently face potentially higher funding costs after the crisis compared to less indirectly exposed banks. We argue that, in the absence of the US financial crisis, more or less indirectly exposed German banks would have experienced similar

[49] Iyer, Peydró, da Rocha-Lopes, and Schoar (2014) and Acabbi, Panetti, Sforza, et al. (2020) use similar exposure measures to study the effect of the interbank market freeze of 2007/08 on firm outcomes in Portugal.

[50] We must restrict lenders to the 80 banks present in the External Position of MFIs dataset. We assume all other banks have zero direct exposure. However, these 80 banks cover 90% of all foreign assets held by the German financial sector.

[51] Since we are interested in the effect of exposure to the US financial crisis on interest rates, we are restricted to banks present in the interest rate sample (ZISTA).

[52] Through the lens of the model in section 2, we interpret the shock to a lender's balance sheet from outside the German market as a shock to T_t^n, since it restricts the bank's ability to provide funding, which is equivalent to stating that the bank has fewer deposits.

changes in funding costs and, consequently, loan interest rates after 2007/2008.

6.2 The Impact of the US Financial Crisis on the German Interbank Market

This section examines the influence of indirect exposure to the US financial crisis on various banking outcomes, such as interest rates, lending decisions, and funding choices. In a scenario where bank funding supply was perfectly elastic, exposed banks should be able to compensate for the funding shortfall caused by directly exposed lenders, resulting in no discernible differential effect of exposure on banking outcomes. Nevertheless, the model presented in section 2 predicts that access to the interbank market plays a significant role in determining funding costs, a notion supported by the stylized fact that interbank positions exhibit considerable persistence.

We hypothesize that banks will raise loan interest rates in response to an adverse funding cost shock, subsequently leading to a decrease in loan demand. Furthermore, if banks are able to partially substitute more expensive interbank sources with deposits and equity, a decrease in interbank borrowing should surpass the reduction in loans. This substitution is evidenced by an increase in the proportion of final loans financed from a bank's own sources, or "own share" in funding. As the efficiency benefits of interbank trade outlined in section 3 directly correlate with the own share, a decline in this measure sheds light on the welfare costs associated with the financial crisis.

To empirically evaluate these model predictions, we investigate the impact of indirect exposure on bank n's outcome variable y_t^n (e.g., final loan interest rate) over time using the following event-study design:

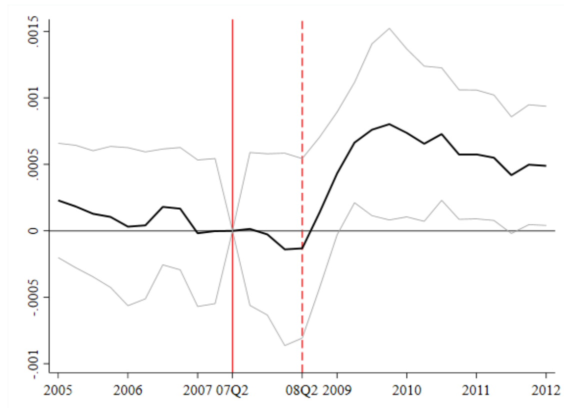
$$\log y_t^n = \rho_n + \mu_t + \sum_{\tau=2004Q4}^{2011Q4} \delta_\tau \left(Exposure_{2006Q1}^{US,n} \times \mu_\tau \right) + X_t^n \beta + u_t^n \ , \tag{16}$$

where ρ_n denotes a bank fixed effect, and μ_t represents a fixed effect for each quarter in our sample. We include a vector of time-varying controls X_t^n such as the shares of various loan products in bank n's aggregate loans (e.g., different borrower types or maturities) and bank n's direct exposure to US MFI assets. By controlling for the composition of a bank's loan portfolio, we aim to eliminate the potential for capturing variation in the average loan rate that arises from adjustments in the types of loan products a bank offers. Furthermore, accounting for a bank's direct exposure to US MFI assets mitigates the possibility of a spurious correlation with its indirect exposure. Importantly, our findings remain robust when excluding these controls (results available upon request). We employ cluster robust standard errors at the bank group-quarter level.[53]

In Figure 5, we present estimates of δ_τ and 95% confidence intervals for four distinct bank variables throughout our sample period. To demonstrate that exposed banks experience limited access to the interbank market following the shock, we plot the coefficients for the log value of

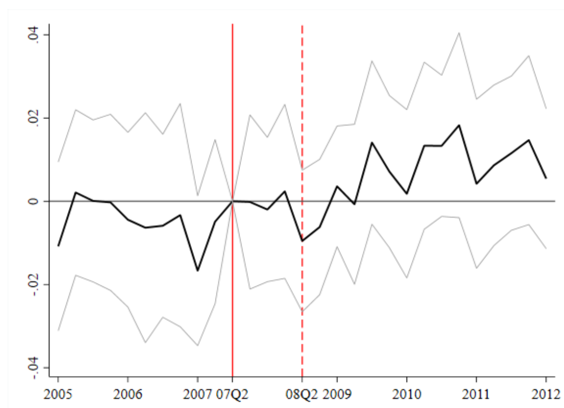
[53] Bank group refers to the classification of banks, such as savings banks, credit banks, cooperative banks, etc.

(a) Interbank borrowing



(b) Interest rate on outstanding final loans

(c) Outstanding final loans



(d) Own share

Figure 5: This event-study examines the impact of indirect exposure to the US financial crisis on borrowing from other German MFIs (upper left), interest rates of final loans (upper right), the quantity of final loans (lower left), and the share of funding from own sources (lower right). Each figure presents coefficients on $Exposure^{US}_{2006Q1} \times Quarter-FE$ and 95% confidence intervals. Solid red vertical lines indicate $2007Q2$, the event quarter immediately preceding the crisis, while dashed red lines represent $2008Q2$, just before the collapse of Lehman Brothers. The regression incorporates quarter fixed effects, bank fixed effects, direct asset exposure to the US, and loan shares of non-MFI and households. These loan shares are further divided into maturities of less than 1 year, between 1 to 5 years, and more than 5 years, as well as separate shares for secured and unsecured mortgages. Robust standard errors are clustered at the bank group-quarter level. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, AUSTA, BISTA, VJKRE, ZISTA, 2004m1 - 2012m1, own calculations.

domestic interbank borrowing in the upper left graph. After the crisis, banks reduce their liabilities with domestic banks by up to 10% per one billion Euro of indirect exposure (over 20% at mean exposure), and this level is maintained thereafter. Importantly, we observe no significant pre-trend in interbank borrowing prior to the crisis.

In line with our interest rate predictions, the point estimates in the upper right figure indicate that interest rates on outstanding loans to firms and consumers rose sharply after the crisis, increasing by up to 8 basis points per 1 billion Euros of indirect exposure and remaining elevated for over 3 years. Our estimated effects are substantial: on average, a bank with the mean indirect exposure of 2.3 billion Euros contracts at an interest rate approximately 20 basis points higher than an otherwise comparable bank with zero exposure. Reassuringly, there is no significant difference in interest rates between more and less exposed banks before the crisis.

The lower left graph investigates the impact of indirect exposure on outstanding loans to firms and consumers. We identify a significant and persistent decline of around 2% per one billion Euro immediately after the crisis, with no notable pre-trend. In accordance with the contagion hypothesis, the funding cost shock originating from banks directly exposed to faltering US assets results in a reduction in credit to the real economy of approximately 5% by banks with mean indirect exposure.

Lastly, in the lower right graph, we display the regression coefficients for the log "own share". This measure is constructed as the difference between final loans and domestic interbank liabilities, divided by final loans for each bank, illustrating the combined effect of exposure on interbank borrowing and loans. If banks were to reduce final loans in the same proportion as interbank borrowing –a proportional shrinking of the balance sheet– we should not observe any impact of indirect exposure on own share. However, after the crisis, more exposed banks substantially decreased interbank borrowing relative to final loans, resulting in an increased reliance on own funding sources. Although not significant at conventional levels, our estimates suggest that banks gradually raised their own share by up to 1.5% per 1 billion Euro, or 3.5% at mean exposure. To gauge the magnitude of this effect, we can compare the mean impact to the increase in the aggregate own share following the Great Recession, as depicted in Figure 2. Between 2008 and 2011, the aggregate own share rose by approximately 10% (from a base of 56%). Therefore, about one-third of the aggregate increase in the own share can be attributed to indirect exposure and the subsequent reduction in interbank activity. In the following analysis, we exploit the funding cost shocks experienced by indirectly exposed banks to estimate the key elasticities of the interbank model.

6.3 Estimation of Banking Sector Elasticities σ and κ

To estimate the demand elasticity σ, we start by taking the logarithms of both sides of bank n's loan demand in equation 2 and replacing aggregate, time-varying variables with a time-fixed effect,

$$\log L_t^n = \mu_t - \sigma \log R_t^{F,n} + \log a_t^n \ , \tag{17}$$

where $\log a_t^n$ captures the time-varying preferences for MFI n at time t. First, we assume $\log a_t^n = \rho_n + \beta' X_t^n + \epsilon_t^n$, where ρ_n is the bank fixed effect and X_t^n represents time-varying controls, as discussed previously. Second, the causal identification of σ requires exogenous variation in interest rates for bank n at time t. We argue that our identification strategy in the preceding section provides variation in loan interest rates that is uncorrelated with demand shocks, ϵ_t^n.

We proceed analogously with our estimation of supply elasticity κ. By substituting equation 10 into 11 for the probability that bank n obtains funds from its own deposits, we establish a structural relationship between interest rates and the "own" share of bank n, i.e., the ratio of M_t^{nn} and loans L_t^n at time t. Specifically,

$$\frac{M_t^{nn}}{L_t^n} = (d_t^{nn})^{-\kappa} (T_t^n)^{-\kappa} \left(\frac{R_t^{F,n}}{R_t^B}\right)^{\kappa} \left(\frac{\sigma - 1}{\sigma}\right)^{\kappa}.$$

Taking logarithms, assuming $d_t^{nn} = 1, \ \forall t$, and collecting all constant and aggregate, time-varying variables into time-fixed effects, we arrive at

$$\log\left(\frac{M_t^{nn}}{L_t^n}\right) = \mu_t + \kappa \log R_t^{F,n} - \kappa \log T_t^n. \tag{18}$$

Equation 18 states that bank n resorts to funding through its own funds with elasticity κ when the cost of funds, including funding from the interbank market, increases. We capture the cost of funds (or interest rate spread) as the interest rate on loans to the real economy net of the bond rate. However, the funding costs of a bank are correlated with unobservable shocks to own funding T_t^n by construction. We assume $\log T_t^n = \rho_n + \beta' X_t^n + v_t^n$, where ρ_n is a bank fixed effect and X_t^n represents time-varying bank-level controls. We interpret v_t^n as an own funds supply shock likely correlated with $R_t^{F,n}$. Equation 18 resembles equation 17, with the difference that we use the own share as the dependent variable. We make the analogous identification assumption as above, specifically that indirect exposure to the US financial crisis is uncorrelated with bank-level deposit shocks v_t^n, and captures variation in $R_t^{F,n}$ coming through the part of bank n's funding costs related to interbank funds. Consequently, we estimate equations 17 and 18 with 2SLS using the interaction of indirect exposure in equation 15 and the post-crisis dummy as an instrument for loan rates $R^{F,n}$.

Panel A of Table 2 presents the first stages (loan rate on the LHS) and reduced forms (loans and own share on the LHS) without controls in columns 1-3 and with controls in columns 4-6. These regressions can be interpreted as parametric versions of our event studies for these outcomes with a single post-crisis dummy. The inclusion of controls has a minimal impact on the size of the coefficients, further supporting our assertion that indirect exposure represents exogenous variation from the perspective of German banks after the crisis. The effect on loan interest rates in column 4 is slightly smaller than the peak of our non-parametric estimate: a bank with mean indirect exposure charges an interest rate that is 14 basis points higher than a comparable bank with zero exposure. The coefficient in column 5 indicates that banks with

Table 2: Estimation of σ and κ

Panel A:	*First Stage and Reduced Forms*					
	(1)	(2)	(3)	(4)	(5)	(6)
	Loan Rate	Loans	Own Share	Loan Rate	Loans	Own Share
Exposure$_{t0}$ $\times$ Post$_t$	0.0004***	-0.0136***	0.0132**	0.0006***	-0.0163***	0.0161***
	(0.0001)	(0.0028)	(0.0060)	(0.0001)	(0.0033)	(0.0058)
R-squared	0.9264	0.9908	0.8613	0.9309	0.9922	0.8642
Mean of Exposure	2.290	2.290	2.290	2.290	2.290	2.290

Panel B:	*2SLS: Instrument for loan rate is Exposure$_{t0}$ $\times$ Post$_t$*					
		$-\sigma$	κ		$-\sigma$	κ
$\log R_t^{F,n}$		-31.20***	30.30**		-27.13***	26.72***
		(8.12)	(13.87)		(5.58)	(9.44)
Observations	3,554	3,554	3,554	3,554	3,554	3,554
Controls	no	no	no	yes	yes	yes
1st Stage F-Stat		12.97	12.97		23.34	23.34

Panel A compares outcomes between $2006Q1$ and $2007Q2$, and $2008Q3$ to $2011Q4$ (post-period) for banks that were more or less indirectly exposed to the US financial crisis. Panel B presents the results of estimating equations 17 and 18 using 2SLS with Exposure$_{t0}$ $\times$ Post$_t$ as an instrument. Initial asset exposure to lenders in the US market is taken in $2006Q1$. Controls include direct asset exposure to the US and loan shares of non-MFI and household loans, each broken down into maturities of less than 1 year, between 1 and 5 years, and more than 5 years, as well as separate shares for secured and unsecured mortgages. All regressions include bank fixed-effects and quarter fixed-effects. Standard errors are clustered at the level of the bank group-quarter. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, AUSTA, BISTA, VJKRE, ZISTA, 2004m12 - 2011m12, own calculations. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

mean exposure reduce lending to the real economy on average by approximately 4 percent in the three years following the Lehman collapse. The coefficient on the own share implies that banks with mean exposure increased their reliance on own funds by 4 percent (or roughly 2 percentage points from a pre-crisis own share of 56%).

Panel B of Table 2 reports IV estimates of σ in columns 2 and 5 and κ in columns 3 and 6. Notably, dropping controls yields very similar elasticities, which bolsters our confidence in the validity of our instrument. The instrument is relevant, as demonstrated by the first-stage F-statistic of around 25 with controls. Our preferred estimate of loan demand elasticity σ is 27.1 in column 5. Reassuringly, this value is quite similar to other estimates from the literature.[54] Regarding our IV estimates of κ, we find that the funding cost shock in the interbank market leads to a significant increase in banks' reliance on own funding sources, with an elasticity of around 26.7 with controls and 30.3 without controls. For our subsequent model analysis, we choose $\kappa = 26.7$ as our preferred estimate for the interbank supply elasticity.

In Table 4, we present coefficients from a pre-trends test. We now include five quarters before 2006Q1 (when we construct the exposure measure) and define the Post dummy for the six quarters up to 2007Q2 (our pre-period in the main estimation). With this setup, we can evaluate whether indirect exposure can predict trends in our outcome variables for the pre-period relative to an earlier period. Consistent with the event-study graphs in Figure 5, our results suggest that there are no significant pre-trends in all variables, except for a small, borderline significant negative trend in loan interest rates.

Table 5 presents several robustness checks. In columns 1-3, we exclude directly exposed banks by restricting the sample to banks that are not present in the AUSTA data. We find similar but slightly stronger results for loans and own share, and weaker effects on interest rates. In columns 4-7, we interact the exposure measure after the crisis with a dummy for banks that are initially net borrowers in the interbank market. We expect net borrowers to be more affected by a funding cost shock since their ability to cover structural deficits on the interbank is now limited (in addition to short-term liquidity management). Indeed, we find larger effects for net borrowers across all outcomes, consistent with our model.

Lastly, we report results from the baseline specification but using lenders' share of US bank assets in total assets (instead of US bank assets in Euros) when constructing the exposure measure.[55] At mean exposure, effects are similar compared to the baseline specification, which indicates that our results are unlikely driven by one or a few very large directly exposed lenders in the network.

6.4 Recovering Model "Wedges"

Having estimated the elasticities σ and κ in the previous subsection, we now turn to characterizing the "wedges" related to the financial shocks $\{a_t^n,\ T_t^n,\ d_t^{ni}\}_{\forall n,i,t}$, for which we specified functional forms in section 2.7. First, we use observed bank-level data on loan interest rates, $\left\{R_t^{F,n}\right\}_{\forall n,t}$, loans to the real economy, $\{L_t^n\}_{\forall n,t}$, shares of funding bank i receives from bank n, $\{\lambda_t^{ni}\}_{\forall n,i,t}$, and the funding share of the central bank for each bank n, $\{\xi_t^{0n}\}_{\forall n,t}$, to recover estimates of the financial shocks for each quarter from the model's equilibrium relationships.

[54]For example, Ulate (2021) estimates loan demand elasticities of 26.6 for the US, 37.4 for the Eurozone, and around 40 for the UK and Japan.

[55]Few large banks might be heavily exposed to US assets in terms of Euros but not necessarily in terms of share of the balance sheet due their overall size. We thank Banu Demir for this suggestion.

We back out estimates for $\{T_t^n\}_{\forall n,t}$ as

$$\hat{T}_t^n = (\lambda_t^{nn})^{-1/\kappa} \left(1 - \xi_t^{0n}\right)^{-1/\kappa} \left(\frac{\sigma}{\sigma - 1}\right)^{-1} R_t^{F,n} ,$$

which is an expression that we obtain after combining equation (10) for the own trade share together with equations (4) and (9). Using the same set of equations but employing the formula for the bilateral trade share between any $\{n, i\}$ pair of banks, we obtain our estimates for $\{d_t^{ni}\}_{\forall n,i}$ as

$$\hat{d}_t^{ni} = \left(\hat{T}_t^n\right)^{-1} (\lambda_t^{ni})^{-1/\kappa} \left(1 - \xi_t^{0i}\right)^{-1/\kappa} \left(\frac{\sigma}{\sigma - 1}\right)^{-1} R_t^{F,i} .$$

Lastly, we recover the $\{a_t^n\}_{\forall n,t}$ shocks by using the CES loan demand from equation (2).

In a second step, we use the time series of the recovered shocks to obtain estimates of their steady-state values $\{a^n,\ T^n,\ d^{ni}\}_{\forall n,i}$, autoregressive coefficient ρ_I, and variance-covariance matrix between shocks, for which we impose a specific structure described in Appendix 3.B.
The only technical difficulty that we have to address relates to the fact that our interest rate sample (ZISTA) covers only between 200 and 240 banks per quarter, as discussed in section 4. The reported banks are selected through stratified sampling, which assigns banks to between 15 and 17 groups using a criterion that combines headquarter state and bank groups in order to capture regional and institutional heterogeneity. The largest banks within each stratum are then selected into the sample.

We construct predicted interest rates for the remaining banks in the main sample by computing a regression of interest rates on observable bank characteristics and detailed balance sheet composition, which we observe for all banks and quarters in our main dataset. We test for sample selection bias and prediction performance by excluding the two smallest ZISTA banks of each stratum (approximately 35 banks per quarter). The R-squared of our prediction is 81.5%, and the average out-of-sample deviation of the predicted versus observed interest rates for the excluded sample is +4 basis points, which suggests that predicting using this selected sample does not create sizable bias for smaller banks outside the sample.

Finally, the remaining macroeconomic parameters are calibrated to reasonable values within the literature's accepted range. Table 6 provides a summary of the selected parameter values.

7 Quantification

7.1 Gains from Financial Integration in Germany

Table 3 reports gains from trade under alternative calibrations of σ and κ elasticities,[56] with values ranging between a maximum of 5.45% and a minimum of -15.9% of steady-state consumption. Our preferred calibration (highlighted in bold) yields a welfare gain from the current in-

[56] The remaining parameters are calibrated to the values estimated in section 6.

terbank market integration of 1.33%. It is important to note that under these values of κ and σ, the total gains are approximately 40% larger than static gains, indicating the significance of second-order welfare gains resulting from a reduction in economic volatility following market integration. Furthermore, when the difference between κ and σ is substantial, dynamic gains can become negative (highlighted in red). In some cases, they become larger than the strictly positive static gains, leading to overall negative gains.

Table 3: Welfare Gains under Alternative Values of σ and κ

Gains from Trade, in %			
		σ	
κ	7	27.1	100
7	5.45	2.10	-15.9
15	3.33	1.83	0.55
26.7	2.18	**1.33**	0.68
100	0.62	0.48	0.32

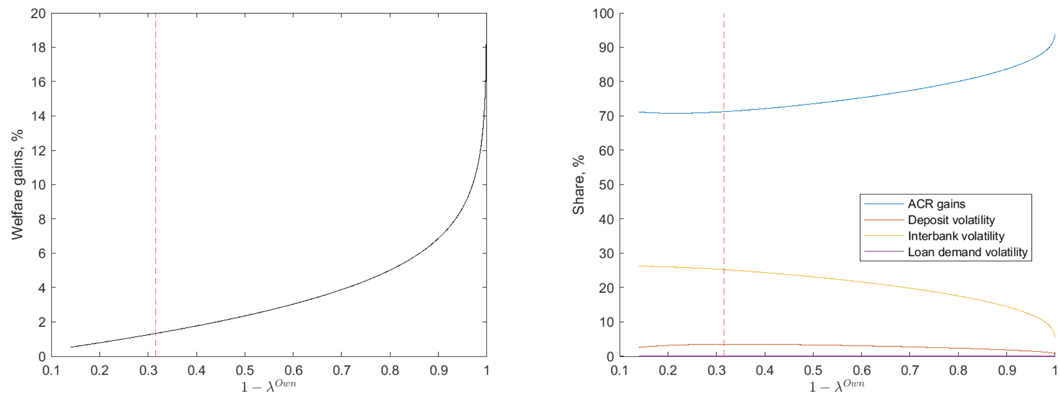
Steady State Gains, in %			
		σ	
κ	7	27.1	100
7	3.74	1.74	3.41
15	2.47	1.25	0.62
26.7	1.72	**0.95**	0.48
100	0.61	0.45	0.26

The table presents estimated gains from interbank trade in Germany under alternative calibrations of the demand elasticities σ and κ. From left to right, the panels display the total gains and the steady-state gains from trade, respectively. Estimates are reported as percentages of steady-state consumption, with our preferred calibration highlighted in bold. Calibrations in which dynamic gains from trade are negative are indicated in red.

Figure 6 presents the gains from trade at different counterfactual levels of interbank market openness, which we calculate by calibrating ϱ in equation (8) to values within the $[0, +\infty)$ range. The limits correspond to zero and infinity (autarky) transaction costs, respectively.[57] Panel (a) demonstrates that the gains from trade range from 1.33% consumption per quarter under the current regime ($\varrho = 1$) to a theoretical maximum of 18% when trade costs are entirely eliminated. Panel (b) plots the share of dynamic gains explained by each component of equation (12). Steady-state gains and lower interbank volatility are the two most significant sources of welfare, accounting for approximately 70% and 30% of the total gains, respectively. Additionally, observe that gains from lower costs of volatility rapidly diminish as banks increase their participation in the interbank market. This finding indicates that most of the benefits from interbank diversification occur at the initial stages of integration.

Lastly, we investigate the costs of the 2007 financial crisis through its impact on the German interbank market, which persistently shrank in size following the onset of the recession (Figure 2). The crisis is such an out-sized, unique contagion event that we treat it separately from the stochastic properties incorporated into the welfare calculations above. Consequently, we assume a structural break in the parameters related to the banking sector $\{T^n, a^n, d^{ni}\}_{\forall n,i}$

[57] Note that changes in ϱ generate proportional changes in the steady-state transaction costs between all banks. Welfare gains under alternative non-proportional integration patterns are not considered in this exercise but can be computed with minor adjustments to the model.



(a) Gains from trade

(b) Decomposition of gains

Figure 6: Gains from interbank trade. Panel 6a presents the gains from trade under the assumption of a proportional reduction or increase in transaction costs between banks. Panel 6b shows the contribution to the gains from trade of the different components in equation (12). A dashed red line marks the current level of interbank integration.

around the event and divide the sample at 2007Q2 and 2008Q3.[58] We estimate a welfare loss of 0.56%, from pre-crisis gains of 1.73% consumption per quarter to 1.17% in the second half of the sample. The welfare loss of 0.56% of consumption is quite sizable (and persistent) compared to estimates in the literature since Lucas (1987)' cost of business cycle calculations, and the loss is of the same order of magnitude as our average static or dynamic gains in Table 3.

7.2 Lender-of-Last-Resort Policy in Germany

We calibrate the model to the German economy under the assumption of a Bundesbank interbank share, denoted in the model as ξ^0, of 3.5%, which is consistent with the pre-crisis average.

As discussed in section 3.2, the gains from the lender-of-last-resort (LoLR) policy can be decomposed into gains from short-term liquidity provision (arising from idiosyncratic intra-quarter funding shocks) and gains associated with the cyclical fluctuations in funding costs (as captured by the banks' credit spreads across quarters). We derive equation (14) with the policy parameter ϖ_2 set to zero (no response to cyclical fluctuations in the credit spread). Consequently, this equation captures welfare gains from liquidity provision, which we find to be 2.5% of consumption per quarter.

Figure 7 quantifies the gains associated with a cyclical response in liquidity provision within the policy parameter space delineated by the pairs $\{\xi^0,\ \varpi_2\}$. In this context, the isoquants demarcate equivalent welfare gains for different parameter combinations. Two significant observations emerge from the figure. First, welfare gains exhibit a monotonic increase with the share of liquidity provided by the lender-of-last-resort (LoLR), ξ^0. It is crucial to interpret this result

[58]We exclude the middle period, 2007Q2 to 2008Q3, to avoid uncertainty surrounding the start of the crisis.

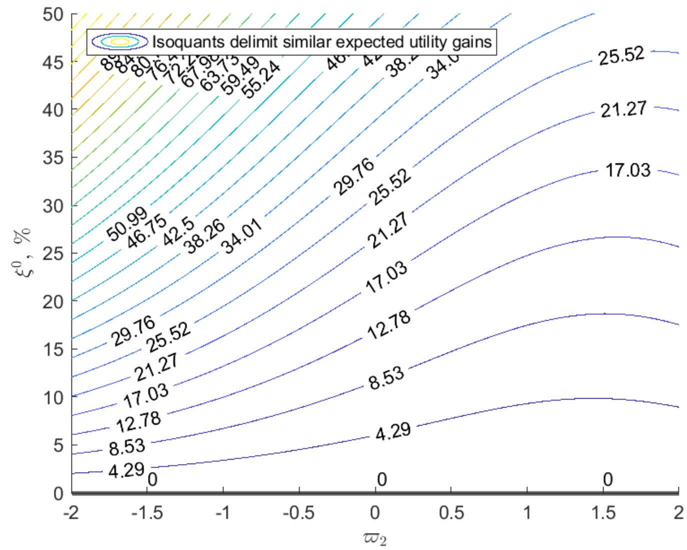


Figure 7: Gains from trade for alternative lender-of-last-resort (LoLR) policy parameter calibrations, own calculations. The y-axis displays the percentage participation of the central bank in the interbank market, while the x-axis represents the responsiveness of the penalty rate to deviations of funding costs from the steady state. Isoquants exhibit constant levels of welfare gains (in percentage) across the parameter space.

cautiously, however, as the model does not incorporate any costs arising from the provision of liquidity by the central bank (e.g., resource misallocation, monitoring costs, moral hazard, etc.). While such costs may be relatively small at lower levels of intervention, they are likely to be relevant in determining the true gains at some of the more extreme points considered in the figure. Second, welfare is enhanced (albeit mildly) when the borrowing penalty imposed by the LoLR exhibits a positive correlation with the credit spread of the banks, as captured by a negative ϖ_2.[59] Although this result appears counterintuitive, it follows from interbank transaction shocks functioning as negative supply shocks to the economy's capacity to produce loans (and hence, capital) within the context of our model. The interaction between interbank transaction shocks and price rigidity creates a positive deviation of the output gap that the lending facility can optimally combat by raising the borrowing penalty. It is worth noting that a more sophisticated model with built-in amplification mechanisms should be capable of reversing this result and producing beneficial countercyclical responses to the credit spread, provided it can generate negative output gap deviations in response to negative supply shocks.[60]

[59] By employing an informed estimate of ϖ_2 at 0.25, derived from the observed evolution of the Bundesbank interbank share during the 2007/08 crisis, the gains from the lender-of-last-resort amount to 2.14% of consumption per quarter. This represents a modest decline in welfare by 0.36% compared to a scenario with an acyclical penalty rate (i.e., $\varpi_2 = 0$).

[60] The reference point chosen for the second-order welfare approximation is also important. Throughout this paper, we maintained a neutral stance with regard to the nature of interbank market shocks, assuming that they constitute a component of the "efficient" economy and, consequently, have an impact on the natural level of output, denoted

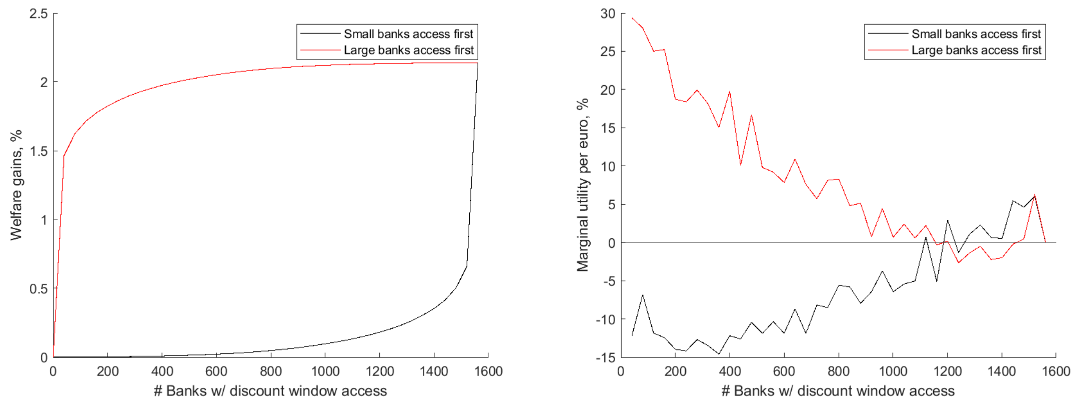
Lastly, we investigate the variation in welfare gains resulting from LoLR policies when granting discount window access to a more limited or extensive set of banks. Our methodology is as follows: initially, we rank the monetary financial institutions (MFIs) in our sample according to their balance sheet sizes. We then calculate households' utility under the assumption that none of the MFIs can borrow from the central bank. Subsequently, we progressively increase the number of banks with discount window access in batches of approximately forty banks at a time, either from smallest to largest or vice-versa. As illustrated in Figure 8a, extending access to a larger subset of banks consistently proves advantageous. However, the majority of the gains from LoLR policies are derived from granting discount window access to the largest MFIs in the sample, with approximately 75% of the total gains accrued to the first hundred largest MFIs. This observation is consistent with the well-established notion of "too big to fail". Changes in the interbank borrowing conditions of the largest banks in the system have the potential to generate aggregate economic fluctuations and, as a result, account for the majority of the welfare gains associated with the central bank's lender-of-last-resort policy. One potential limitation with this finding is that larger banks typically necessitate a greater amount of liquidity when accessing the discount window. Therefore, a more suitable question to assess the efficacy of liquidity provision to different participants would be to inquire about the marginal utility per Euro of liquidity provided. We construct such a measure as follows:

$$\mathbb{W}^{(j)} \equiv \frac{\mathbb{G}^{(j)} - \mathbb{G}^{(j-1)}}{M^{0(j)} - M^{0(j-1)}} ,$$

where $\mathbb{G}^{(j)}$ and $M^{0(j)}$ represent, respectively, the expected gains from LoLR and the total steady-state liquidity provided by the central bank after granting discount window access up to (and including) batch j of banks. Figure 8b displays a normalized version of this variable, $\frac{\mathbb{W}^{(j)}}{\mathbb{W}^{(\text{all})}} - 1$, which expresses the marginal returns per Euro of batch (j) as a percentage of the marginal returns of the final batch encompassing all banks, denoted by the (all) superscript. Observing the red line in Figure 8b, we find that the marginal welfare returns per Euro of liquidity provided to the largest group of banks are up to 30% greater than those of the final batch of smaller banks. With the alternative order of discount window access depicted by the black line, we obtain a similarly consistent result, in which the marginal welfare returns per Euro to small banks are 10% lower than those of large banks. This outcome is in line with the centrality of large banks in the interbank network, as previously discussed in relation to the number of unique connections presented in Figure (10).

Considering the broader implications, our findings suggest that potentially significant welfare benefits could be realized by extending discount window access to financial entities that have traditionally been excluded from it, such as investment funds and insurers. We defer the exploration of such questions to future research.

as y_t^n. Conversely, if these shocks were excluded from the efficient economy (for instance, by attributing their origins to market inefficiencies or irrational behavior), it would be possible to generate negative deviations in the output gap without necessitating further modifications to the model.



(a) Gains from LoLR (b) Marginal Gains

Figure 8: Welfare gains from expanded discount window access, own calculations. The x-axis represents the number of banks with access to the discount window in the counterfactual scenario. Banks are arranged by balance sheet size, with smaller banks granted access either first (black line) or last (red line). Owing to confidentiality constraints, the results assume an expansion of the discount window in increments of approximately forty banks at a time. Panel 8a illustrates welfare gains (in percentage) compared to the no-access counterfactual. Panel 8b displays the relative marginal utility per Euro in relation to the last group of banks granted access (in percentage).

8 Conclusion

In this paper, we investigate the welfare implications of interbank market integration and the role of lender-of-last-resort policy in mitigating the costs of financial volatility in the banking sector. By incorporating a granular representation of the interbank market into a DSGE model and combining it with proprietary data on the universe of German banks, we quantify the static and dynamic gains from interbank trade at various levels of integration and under alternative lender-of-last-resort policies. Using a difference-in-difference framework, we show that the US financial crisis propagated to German banks and the real economy through banks' lending networks, resulting in higher funding costs, reduced credit, and a persistent decline in interbank market volume, as our model predicts. From the perspective of our model, market integration generates first-order welfare gains from enhanced efficiency in resource allocation but involves second-order trade-offs between the benefits of diversification and the drawbacks of increased counterparty risk exposure. We estimate gains to be approximately 1.33% of consumption per quarter, stemming from a combination of efficiency gains in fund allocation across the bank network and reduced volatility through diversification of banks' funding sources, which, in practice, outweigh the costs of heightened risk exposure. Moreover, we find that lender-of-last-resort liquidity provision is an effective tool for reducing steady-state interest rate spreads even at low levels of intervention, but has limited scope for mitigating the costs of financial market fluctuations across the business cycle.

Lastly, we anticipate that our model will prove valuable for exploring a range of different topics, which we leave for future research. Among these, we consider the examination of lender-of-last-resort policies at the zero lower bound and research on international processes of financial integration, such as those following the creation of the European Union and the adoption of the Euro, as particularly intriguing. We also believe that a fruitful avenue for future work lies in linking our reduced-form results on the bank side with data on households and firms to directly assess the impact of interbank markets on the real economy.

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Figures & Tables

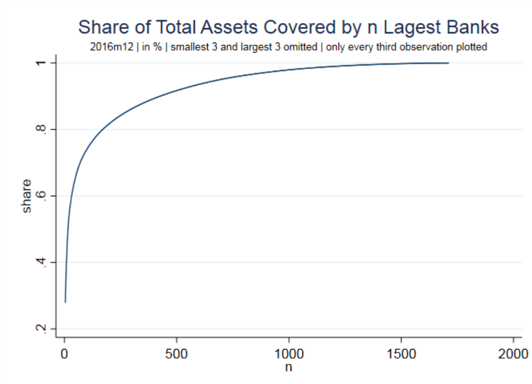
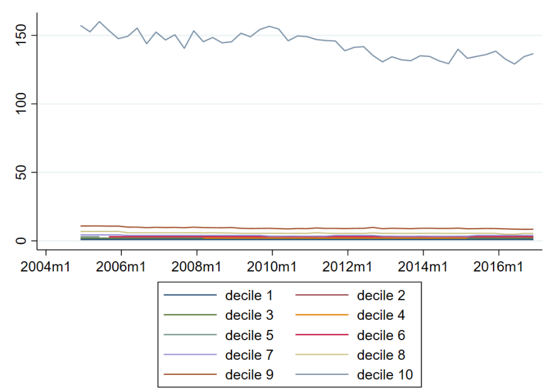


Figure 9: Cumulative share of total MFI assets by the n largest MFIs. The smallest and largest three MFIs are omitted, and only every third observation is plotted due to confidentiality requirements. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA, 2016m12, own calculations.

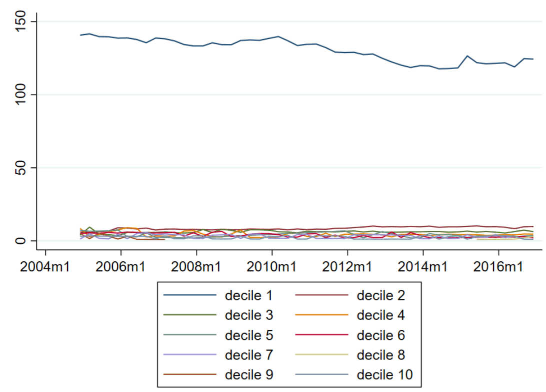
Table 4: Pre-trends estimation of σ and κ

	(1) Loan Rate	(2) Loans	(3) Own Share	(4) Loan Rate	(5) Loans	(6) Own Share
$Exposure_{t0} \times Post_{t0}$	-0.0000	0.0027	-0.0061*	-0.0001*	-0.0021	-0.0052
	(0.0000)	(0.0042)	(0.0032)	(0.0001)	(0.0026)	(0.0032)
Observations	1,768	1,768	1,768	1,768	1,768	1,768
R-squared	0.9552	0.9963	0.9599	0.9598	0.9969	0.9613
Controls	no	no	no	yes	yes	yes
Mean of Exposure	2.290	2.290	2.290	2.290	2.290	2.290

The regression compares outcomes between $2004Q4$ to $2005Q4$ and the pre-period in the main regression ($2006Q1$ until $2007Q2$) for banks with varying degrees of indirect exposure to the US financial crisis. The initial asset exposure to lenders in the US market is taken in $2006Q1$. Controls include direct asset exposure to the US and loan shares of non-MFI and household loans, each broken down into maturities of less than 1 year, between 1 and 5 years, and more than 5 years, as well as separate shares for secured and unsecured mortgages. All regressions include bank fixed-effects and quarter fixed-effects. Standard errors are clustered at the level of the bank group-quarter. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, AUSTA, BISTA, VJKRE, ZISTA, 2004m12 - 2007m6, own calculations. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$



(a) By number of connections



(b) By bank size

Figure 10: Average number of distinct interbank funding sources, by deciles. Figure 10a constructs deciles based on the number of distinct interbank funding sources. Figure 10b defines deciles with respect to the total asset size of the MFIs. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA and Credit Registry, 2004m12 - 2018m12, own calculations.

Table 5: Robustness Checks for estimating σ and κ

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Loan Rate	Loans	Own Share	Loan Rate	Loans	Own Share	Borrowing	Loan Rate	Loans	Own Share
$\text{Exposure}_{t0} \times \text{Post}_t$	0.0002	-0.0314***	0.0410***	0.0004***	-0.0139***	0.0035	-0.0521***	0.2016***	-10.8128***	4.6847***
	(0.0002)	(0.0105)	(0.0114)	(0.0001)	(0.0036)	(0.0051)	(0.0173)	(0.0421)	(2.4522)	(1.5819)
Net Borrower$\times$										
$\text{Exposure}_{t0} \times \text{Post}_t$				0.0003*	-0.0037	0.0193***	-0.0380***			
				(0.0001)	(0.0026)	(0.0035)	(0.0087)			
Observations	2,381	2,381	2,381	3,554	3,554	3,554	3,551	3,554	3,554	3,554
R-squared	0.9360	0.9839	0.8690	0.9311	0.9922	0.8651	0.9537	0.9309	0.9923	0.8641
Controls	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Robustness	Drop	Drop	Drop	By Net	By Net	By Net	By Net	Scaled	Scaled	Scaled
	AUSTA	AUSTA	AUSTA	Borrower	Borrower	Borrower	Borrower	Exposure	Exposure	Exposure
Mean of Exposure	2.280	2.280	2.280	2.290	2.290	2.290	2.290	0.00986	0.00986	0.00986

The regression compares outcomes between $2004Q4$ to $2005Q4$ and the pre-period in the main regression ($2006Q1$ until $2007Q2$) for banks with varying degrees of indirect exposure to the US financial crisis. The initial asset exposure to lenders in the US market and net borrower position are taken in $2006Q1$. Controls include direct asset exposure to the US and loan shares of non-MFI and household loans, each broken down into maturities of less than 1 year, between 1 and 5 years, and more than 5 years, as well as separate shares for secured and unsecured mortgages. All regressions include bank fixed-effects and quarter fixed-effects. Standard errors are clustered at the level of the bank group-quarter. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, AUSTA, BISTA, VJKRE, ZISTA, 2004m12 - 2007m6, own calculations. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 6: Parameter Calibration

Parameter	Value	Description	Source
η	1	Frisch labor supply elasticity	Standard
β	0.99	Discount factor	Standard
ϵ	7	Elasticity of substitution intermediate output	Standard
θ	0.55	Calvo price stickiness	Standard
α	0.4	Capital share in production	Aggregate data
σ	27.1	Firm's loan elasticity of demand	Estimation, Sec. 6
κ	26.7	Interbank loan elasticity of demand	Estimation, Sec. 6
$\bar{\Pi}$	0	Target inflation rate	Standard
γ_π	2.5	Taylor rule inflation response	Standard
γ_y	1.5	Taylor rule output gap response	Standard
ϖ_1	0.12	Fixed penalty rate	Match 3.5% pre-crisis CB trade share
ϖ_2	0.25	Variable penalty rate responsiveness	Educated guess
ρ_I	0.77	Persistence interbank shocks	Estimation, Sec. 6
ζ_T	0.077	Covariance depositor preferences shock	Estimation, Sec. 6
$\zeta_{I,B}$	0.052	Covariance interbank transactions shock, same borrower	Estimation, Sec. 6
$\zeta_{I,L}$	0.52	Covariance interbank transactions shock, same lender	Estimation, Sec. 6
$\zeta_{I,X}$	0.025	Covariance interbank transactions shock, different lender and borrower	Estimation, Sec. 6
$\sigma_a \cdot (1 - \zeta_a)$	0.001	Standard deviation firm-loan demand shock and covariance, joint	Estimation, Sec. 6
σ_T	0.028	Standard deviation depositor preferences shock	Estimation, Sec. 6
σ_I	0.04	Standard deviation interbank transactions shock	Estimation, Sec. 6

The table presents the baseline parameter values used to calibrate the model in Section 6. The last column indicates the reference source for parameter calibration.

Online Appendix (Not For Publication)

Appendix 1 Detailed Model Derivation

Representative Household The instantaneous utility function for the household, as presented in Section 2.2, can be expressed as follows:

$$U_t = \log(X_t) - \left(\frac{\eta}{\eta+1}\right)\int_0^1 N_{t,\tau}^{1+1/\eta}\,\mathrm{d}\tau\ .$$

In this equation, X_t represents the consumption of the composite good, and $N_{t,\tau}$ denotes the aggregate labor supply of the household. The budget constraint for the representative household in period (t,τ) is given by:

$$\upsilon_{t,\tau}: \qquad C_{t,\tau} + \frac{\sum_{n=1}^{N} D_{t,\tau}^{n}}{P_t} + \frac{B_{t,\tau}}{P_t} = \frac{\sum_{n=1}^{N}(1+\varsigma_D)\cdot R_{t-1,\tau}^{D,n} D_{t-1,\tau}^{n}}{P_t} + \frac{R_{t-1}^{B} B_{t-1,\tau}}{P_t} + \int_0^1 \frac{W_t(\nu) N_{t,\tau}(\nu)}{P_t}\,\mathrm{d}\nu + \frac{\Upsilon_{t,\tau}}{P_t}\ . \tag{A.1}$$

In the above constraint, $B_{t,\tau}$ and $R_{t,\tau}^{B}$ correspond to one-period government bonds and the interest rate paid on them, respectively. The interest rate paid on deposits by bank n is denoted by $R_{t,\tau}^{D,n}$. The wage paid by industry ν is represented by $W_t(\nu)$, while P_t is the aggregate price index. Firm and bank profits, which are lump-sum transferred to the agent, are symbolized by $\Upsilon_{t,\tau}$. Additionally, ς_D stands for a subsidy on bank deposits, and $\upsilon_{t,\tau}$ serves as the Lagrange multiplier associated with the constraint.

The representative household seeks to maximize its expected present discounted utility, subject to the budget constraint expressed in equation (A.1). The first-order conditions of this optimization problem are given by:

$$C_{t,\tau}: \qquad \upsilon_{t,\tau} = X_t^{-1}\ , \tag{A.2}$$

$$N_{t,\tau}(\nu): \qquad N_{t,\tau}(\nu)^{1/\eta} = \upsilon_t \frac{W_t(\nu)}{P_t}\ , \qquad \forall \nu\ , \tag{A.3}$$

$$B_{t,\tau}: \qquad 1/R_t^{B} = \beta E_t\left[\frac{\upsilon_{t+1,\tau}}{\upsilon_{t,\tau}\Pi_{t+1}}\right]\ , \tag{A.4}$$

$$D_{t,\tau}^{n}: \qquad 1/R_{t,\tau}^{D,n} = (1+\varsigma_D)\cdot\beta E_t\left[\frac{\upsilon_{t+1,\tau}}{T_t^{n} z_{t,\tau}^{n}\upsilon_{t,\tau}\Pi_{t+1}}\right]\ , \qquad \forall n\ . \tag{A.5}$$

By combining equations (A.4) and (A.5), we derive the following relationship between the interest rates of deposits and bonds:

$$R_{t,\tau}^{D,n} = (1+\varsigma_D)^{-1}\cdot T_t^{n}\cdot z_{t,\tau}^{n}\cdot R_t^{B}\ . \tag{A.6}$$

Firms The assumption of complete depreciation and immediate capital accumulation from investment, discussed in Section 2.3, entails that $K_{t,\tau}^{n}(\nu) = I_{t,\tau}^{n}(\nu)/P_t\ ,\ \forall\, n$, with $I_t^{n}(\nu)$ representing the investment in capital of type n. Firms finance their investment at each moment τ through credit obtained from banks. The model includes N distinct banks, each specializing in providing loans $L^n_{t,\tau}(\nu)$ for a different type of capital. Loans are repaid after one quarter at a gross interest rate of $R^{F,n}_t$, with firms subject to the investment constraint $I^n_{t,\tau}(\nu) \leq L^n_{t,\tau}(\nu)$, $\forall\, n, t, \tau$.

A representative, perfectly competitive firm combines intermediate products into a final good according to the following equation:

$$Y_t = \left[\int_0^1 Y_t(\nu)^{\left(\frac{\epsilon-1}{\epsilon}\right)} \,\mathrm{d}\nu\right]^{\left(\frac{\epsilon}{\epsilon-1}\right)} ,$$

where $\epsilon > 1$ denotes the elasticity of substitution between varieties. Individual demand for intermediates is given by:

$$Y_t(\nu) = \left(\frac{P_t(\nu)}{P_t}\right)^{-\epsilon} Y_t \ ,$$

with $P_t(\nu)$ representing the price of intermediate ν and $P_t = \left[\int_0^1 P_t(\nu)^{1-\epsilon}\,\mathrm{d}\nu\right]^{\frac{1}{1-\epsilon}}$ as the aggregate price index.

Intermediate producers exhibit sticky prices à la Calvo (1983), resetting their prices at the beginning of a quarter with probability $1-\theta$. In equilibrium, all firms reset to the same optimal price within a given period, denoted by $P^\star_t$. The previous equation can be recursively expressed as:

$$P_t^{1-\epsilon} = (1-\theta)\cdot(P^*_t)^{1-\epsilon} + \theta\cdot(P_{t-1})^{1-\epsilon} \ . \tag{A.7}$$

Intermediate firm ν aims to maximize the following present discounted stream of profits:

$$\sum_{j=0}^{\infty} E_t\left[Q_{t,t+j}\int_0^1 (1+\varsigma_F)P_{t+j}(\nu)Y_{t+j,\tau}(\nu) - W_{t+j}(\nu)N_{t+j,\tau}(\nu) - \sum_{n=1}^{N} R^{F,n}_{t+j-1}L^n_{t+j-1}(\nu)\,\mathrm{d}\tau\right] ,$$

where $Q_{t,t+j} = \beta^j \dfrac{X_t}{X_{t+j}\Pi_{t+j}}$ represents the firm's stochastic discount factor between periods t and $t+j$, and ς_F is a government production subsidy. By minimizing firm ν's production costs with respect to labor and loans, we obtain the following demand for inputs:

$$N_t(\nu) = (1-\alpha)\cdot\frac{Y_t(\nu)}{A_t}\left(\frac{\widetilde{R}^F_t}{W_t(\nu)/P_tA_t}\right)^{\alpha} , \tag{A.8}$$

$$\frac{L_t(\nu)}{P_tA_t} = \alpha\cdot\frac{Y_t(\nu)}{A_t}\left(\frac{\widetilde{R}^F_t}{W_t(\nu)/P_tA_t}\right)^{-(1-\alpha)} ,$$

$$L^n_t(\nu) = a^n_t\left(\frac{R^{F,n}_t}{R^F_t}\right)^{-\sigma} L_t(\nu) \ , \tag{A.9}$$

where α is the share of capital in the production function, $A_t \equiv exp\left(u_t^A\right)$ is the aggregate total factor productivity, and $\widetilde{R}_t^{F,n} = E_t\left[Q_{t,t+1}\right] \cdot R_t^{F,n}$ is the expected discounted gross rate on a loan from bank n and $R_t^F = \left[\sum_{n=1}^N a_t^n \left(R_t^{F,n}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is the aggregate gross rate index. Maximizing the present discounted stream of profits also yields the optimal reset price at time t, $P_t^\star$:

$$\frac{P_t^*}{P_t} = \frac{E_t\left[\sum_{j=0}^\infty \theta^j Q_{t,t+j}\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon+1} Y_{t+j}\left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right)\left(\frac{MC_{t+j|t}(\nu)}{P_{t+j}}\right)\right]}{E_t\left[\sum_{j=0}^\infty \theta^j Q_{t,t+j}\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon} Y_{t+j}\right]}\ , \tag{A.10}$$

where subindex $t+j|t$ denotes the value of a variable conditional on the firm having last reset its price at period t, and $MC_{t+j|t}(\nu)/P_t = \left(\widetilde{R}_{t+j}^F\right)^\alpha \left(\frac{W_{t+j|t}(\nu)}{P_{t+j}A_{t+j}}\right)^{1-\alpha}$ is the real marginal cost of production, with $\widetilde{R}_t^F = E_t\left[Q_{t,t+1}\right] \cdot R_t^F$. Over the period, the aggregate firm profits are given by:

$$\Upsilon_t^F = (1+\varsigma_F)P_tY_t - \int_0^1 W_t(\nu)N_t(\nu)\ \mathrm{d}\nu - \sum_{n=1}^N R_{t-1}^{F,n} L_{t-1}^n\ .$$

Banks Each bank conducts three distinct activities: obtaining deposits from the representative household, providing credit to firms, and lending funds to one another in the interbank market. For the sake of exposition, we assume that each bank is divided into two Divisions, each responsible for performing a different set of these activities. The Loan Divisions provide credit to firms and secure the necessary funding through internal funds or interbank loans. The Deposit Divisions procure deposits from the representative household and distribute them to the Loan Divisions via the interbank market or internal funds transfer.

Loan Division: Loan Division n seeks to maximize expected profits:

$$\begin{aligned} &\int_0^1 (1+\varsigma_B)R_t^{F,n}L_{t,\tau}^n - R_{t,\tau}^{I,n}M_{t,\tau}^n\ \mathrm{d}\tau\ , \\ \text{where:}\quad &R_{t,\tau}^{I,n} = \min_i\left\{R_{t,\tau}^{I,in}\right\}\ , \\ &M_{t,\tau}^n = M_{t,\tau}^{in},\quad i_{t,\tau}(n) = \arg_j\min\left\{R_{t,\tau}^{I,jn}\right\}\ , \end{aligned} \tag{A.11}$$

subject to the constraints in (3). ς_B represents a government subsidy to banks lending to firms, and variable $R_{t,\tau}^{I,n}$ denotes the rate at which interbank loans (or own funds) at point τ are obtained by bank n. Banks are aware of their firm loan demands, as specified by equation (A.9), and act as monopolistic competitors, taking the aggregate gross rate index R^F as given. We assume that rates on firm loans are sticky *within* the continuum and can only be reset at the beginning of each period. Conversely, interbank rates are fully flexible and reflect the capacity to provide funds of the emitting bank at each moment τ. Solving the maximization problem yields the optimal interest rate on firm loans as a constant mark-up over the average cost of funds:

$$R_t^{F,n} = \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma - 1} \right) R_t^{I,n} \ , \qquad R_t^F = \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma - 1} \right) R_t^I \ , \tag{A.12}$$

where we define $R_t^{I,n} \equiv \int_0^1 R_{t,\tau}^{I,n} \, \mathrm{d}\tau$ and $R_t^I = \left[\sum_{n=1}^N a_t^n \cdot \left(R_t^{I,n} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$.

Deposit Division: Deposit Divisions acquire deposits from the representative household and transform them into internal funding or interbank loans for other banks. The quantity of funds that bank n can provide is expressed as:

$$\sum_{i=1}^N d_t^{ni} \cdot M_{t,\tau}^{ni} = D_{t,\tau}^n \ , \tag{A.13}$$

subject to $M_{t,\tau}^{ni} \geq 0, \ D_{t,\tau}^n \geq 0, \ \forall n, i, t, \tau$. The variables $d_t^{ni} \geq 1$ denote transaction costs associated with transferring funds from bank n to i. These costs encompass screening, enforcement, or other expenses related to a transaction. We normalize the transaction costs between Divisions of the same bank to one, i.e., $d_t^{nn} = 1, \ \forall n, t$.

The markets for interbank loans and deposits are perfectly competitive, with banks acting as price takers. The expected profits of Deposit Division n are given by:

$$\int_0^1 \sum_{i=1}^N R_{t,\tau}^{I,ni} M_{t,\tau}^{ni} - R_{t,\tau}^{D,n} D_{t,\tau}^n \ \mathrm{d}\tau \ .$$

Upon solving the optimization problem, the interest rate charged by bank n to i at moment τ is:

$$R_{t,\tau}^{I,ni} = d_t^{ni} \cdot R_t^{D,n} = (1+\varsigma_D)^{-1} \cdot d_t^{ni} \cdot T_t^n \cdot z_{t,\tau}^n \cdot R_t^B \ , \tag{A.14}$$

where equation (A.6) is employed to obtain the final equality.

Central Bank As outlined in Section 2.5, the central bank determines the policy and lending facility rates. Furthermore, we assume that any profits generated by the central bank's lending facility are returned to the representative household through a lump-sum transfer:

$$\Upsilon_t^{CB} = \sum_{n=1}^N \int_0^1 R_{t-1,\tau}^{I,0n} M_{t-1,\tau}^{0n} \ d\tau \ .$$

Banking sector aggregation By incorporating equations (A.14) and (6) into equation (A.11), we derive the distribution of the interbank rate paid by bank n:

$$\widetilde{R}^{I,n}_{t,\tau} \sim W\left(\frac{\widetilde{R}^{I,n}_t}{\Gamma\left(1+1/\kappa\right)}, \kappa\right) \quad , \quad \widetilde{R}^{I,n}_t = \Phi^n_t \cdot \left[1 + e^{-\kappa\varpi_1} \cdot \left(\frac{\Phi^n_t}{\Phi^n}\right)^{\kappa\varpi_2}\right]^{-1/\kappa} ,$$

$$\Phi^n_t = \left[\sum_{i=1}^{N} \left((1+\varsigma_D)^{-1} \cdot d^{in}_t \cdot T^i_t\right)^{-\kappa}\right]^{-1/\kappa} ,$$

where we employ the property that the minimum of a collection of Weibull random variables also follows a Weibull distribution.

Transaction volumes and Deposits We introduce ξ^{0i}_t to represent the proportion of funding that bank i acquires from the central bank:

$$\xi^{0i}_t = \left[1 + e^{\kappa\varpi_1} \cdot \left(\frac{\Phi^i_t}{\Phi^i}\right)^{-\kappa\varpi_2}\right]^{-1} .$$

The transaction volume between any pair of banks can be expressed as:

$$M^{ni}_t = \begin{cases} \xi^{0i}_t \cdot L^i_t \, , & \text{if } n = 0, \\ \left(1 - \xi^{0i}_t\right) \cdot \lambda^{ni}_t \cdot L^i_t \, , & \text{otherwise,} \end{cases} \qquad \text{where: } \lambda^{ni}_t = \left(\frac{(1+\varsigma_D)^{-1} \cdot d^{ni}_t T^n_t}{\Phi^{I,i}_t}\right)^{-\kappa} .$$

Variable λ^{ni}_t denotes the share of (non-central bank) borrowing that bank i obtains from bank n. A detailed interpretation of these results is provided in Section 2.8.

By integrating equation (A.13) along the continuum and across banks, we derive the following expressions for aggregate bank deposits:

$$D^n_t = \sum_{i=1}^{N} d^{ni}_t \cdot M^{ni}_t \, ,$$

$$D_t + \sum_{n=1}^{N} M^{0n}_t = \sum_{n=1}^{N} L^n_t + \sum_{n=1}^{N}\sum_{i=1}^{N} \left(d^{ni}_t - 1\right) \cdot M^{ni}_t \, .$$

The latter equation demonstrates that, in equilibrium, aggregate deposits and central bank money correspond to the total amount of loans plus the interbank transaction costs. Lastly, the aggregate banking sector profits over period t can be described as:

$$\Upsilon^B_t = \sum_{n=1}^{N} R^{F,n}_{t-1} L^n_{t-1} - \sum_{n=1}^{N} \int_0^1 R^{D,n}_{t-1,\tau} D^n_{t-1,\tau} \, \mathrm{d}\tau - \sum_{n=1}^{N} \int_0^1 R^{I,0n}_{t-1,\tau} M^{0n}_{t-1,\tau} \, d\tau \, .$$

Government In this model, the government offers subsidies to firms, banks, and depositors, which are financed through lump-sum taxation imposed on the representative household. The expression for government transfers can be given by:

$$\Upsilon_t^G = -\left[\varsigma_F \cdot P_t Y_t + \varsigma_B \cdot \sum_{n=1}^{N} R_{t-1}^{F,n} L_{t-1}^n + \varsigma_D \cdot \sum_{n=1}^{N} R_{t-1}^{D,n} D_{t-1}^n\right] .$$

Market Clearing The total transfers to the representative household can be expressed as:

$$\Upsilon_t \equiv \Upsilon_t^F + \Upsilon_t^B + \Upsilon_t^{CB} + \Upsilon_t^G = P_t Y_t - \int_0^1 W_t(\nu) N_t(\nu)\, \mathrm{d}\nu - (1+\varsigma_D) \cdot \sum_{n=1}^{N} \int_0^1 R_{t-1,\tau}^{D,n} D_{t-1,\tau}^n \, \mathrm{d}\tau .$$

By aggregating the representative household budget constraint (A.1) over the τ continuum and utilizing the previous expression, we derive the following aggregate market clearing condition:

$$C_t + \frac{D_t}{P_t} = Y_t . \tag{A.15}$$

Aggregation By employing equations (A.2), (A.3), (A.8), and (A.12), firm-specific marginal costs can be expressed as a function of aggregate variables:

$$\frac{MC_{t+j|t}(\nu)}{P_{t+j}} = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{X_{t+j}}{A_{t+j}}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_{t+j}}{A_{t+j}}\right)^{\frac{1-\alpha}{\eta+\alpha}} \left(\widetilde{R}_{t+j}^I\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{P_t^*}{P_{t+j}}\right)^{-\left(\frac{\epsilon(1-\alpha)}{\eta+\alpha}\right)} . \tag{A.16}$$

In a similar manner, loan and labor demand can be integrated across the continuum of firms to obtain:

$$\frac{L_t}{A_t P_t} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \left(\frac{X_t}{A_t}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_t}{A_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\widetilde{R}_t^I\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \Delta_t , \tag{A.17}$$

$$N_t = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{X_t}{A_t}\right)^{(1-\alpha)\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{X_t}{Y_t}\right)^{-\left(\frac{\eta}{\eta+\alpha}\right)} \left(\widetilde{R}_t^I\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}} ,$$

where Δ_t denotes a measure of price dispersion that can be recursively defined as:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta \Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \Delta_{t-1} .$$

Substitute equation (A.16) and the expressions for Q_{t+j} into the optimal resetting price equation (A.10):

$$\left(\frac{P_t^*}{P_t}\right)^{1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)} = \frac{E_t\left[\sum_{j=0}^{\infty} (\theta\beta)^j \, (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right) \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{X_{t+j}}{A_{t+j}}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{Y_{t+j}}{A_{t+j}}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\widetilde{R}_{t+j}^I\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]}{E_t\left[\sum_{j=0}^{\infty} (\theta\beta)^j \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon-1} \left(\frac{X_{t+j}}{A_{t+j}}\right)^{-1} \left(\frac{Y_{t+j}}{A_{t+j}}\right)\right]} .$$

This expression can be simplified as:

$$\frac{P_t^*}{P_t} = \left(\frac{F_t}{H_t}\right)^{1/\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} ,$$

where

$$F_t = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right) \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{X_t}{A_t}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\widetilde{R}_t^I\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta \cdot E_t \left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} F_{t+1}\right] \tag{A.18}$$

$$H_t = \left(\frac{X_t}{Y_t}\right)^{-1} + \theta\beta \cdot E_t \left[\Pi_{t+1}^{\epsilon-1} H_{t+1}\right] .$$

Applying equation (A.7) to the previous equations yields the following equilibrium conditions:

$$\frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} ,$$

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \cdot \Delta_{t-1} .$$

Utilizing equation (A.15), we derive an expression for the composite good as:

$$X_t = Y_t - \sum_{n=1}^{N} \int_0^1 T_t^n \cdot z_{t,\tau}^n \frac{D_{t,\tau}^n}{P_t} \, \mathrm{d}\tau .$$

In the previous equation, the deposits expression is related to the model's aggregate variables as follows:

$$\sum_{n=1}^{N} \int_0^1 T_t^n \cdot z_{t,\tau}^n \frac{D_{t,\tau}^n}{P_t} \, \mathrm{d}\tau = \frac{1}{(1+\varsigma_D)^{-1}} \sum_{n=1}^{N}\sum_{i=1}^{N} \int_0^1 \widetilde{R}_{t,\tau}^{I,ni} \frac{M_{t,\tau}^{ni}}{P_t} \, \mathrm{d}\tau = \frac{1-\xi_t^0}{(1+\varsigma_D)^{-1}} \cdot \widetilde{R}_t^I \cdot \frac{L_t}{P_t} ,$$

where $\xi_t^0 = \sum_{i=1}^N s_t^i \cdot \xi_t^{0i}$ denotes a weighted share of the funding obtained from the central bank lending facility, and $s_t^i = a_t^i \cdot \left(\widetilde{R}_t^{I,i}/\widetilde{R}_t^I\right)^{1-\sigma}$ represents the share of loans to firms supplied by bank i. By incorporating the aggregate loan demand in equation (A.17) and the previous expressions, we obtain:

$$\frac{X_t}{Y_t} = 1 - \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \cdot \left(\frac{X_t}{Y_t}\right)^{-\left(\frac{1-\alpha}{\eta+\alpha}\right)} \left(\frac{X_t}{A_t}\right)^{(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\widetilde{R}_t^I\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{1-\xi_t^0}{(1+\varsigma_D)^{-1}}\right) \Delta_t . \tag{A.19}$$

The real money balances lent by the central bank to bank i are:

$$\frac{M_t^{0i}}{P_t} = \xi_t^{0i} \cdot \frac{L_t^i}{P_t} = s_t^i \xi_t^{0i} \cdot \frac{L_t}{P_t} .$$

The total real money balances lent by the central bank are:

$$\frac{M_t^0}{P_t} = \sum_{i=1}^{N} \frac{M_t^{0i}}{P_t} = \xi_t^0 \cdot \frac{L_t}{P_t} .$$

Flexible Price Equilibrium We define the flexible equilibrium of the economy as the one in which all firms can reset prices every quarter ($\theta = 0$). By combining equations (A.18)-(A.19), we derive the following expressions:

$$X_t^n = (1-\alpha)^{-\left(\frac{1}{\eta+1}\right)} \left[1 - \alpha \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon - 1}\right)^{-1} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma - 1}\right)^{-1} \left(\frac{1-\xi_t^0}{(1+\varsigma_D)^{-1}}\right)\right]^{\frac{1}{\eta+1}} \cdot \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon - 1}\right)^{-\left(\frac{1}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta+1}\right)} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma - 1}\right)^{-\left(\frac{\alpha}{1-\alpha}\right)} A_t \cdot \left(\widetilde{R}_t^I\right)^{-\left(\frac{\alpha}{1-\right.}}$$

$$\frac{X_t^n}{Y_t^n} = 1 - \alpha \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon - 1}\right)^{-1} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma - 1}\right)^{-1} \left(\frac{1-\xi_t^0}{(1+\varsigma_D)^{-1}}\right),$$

where the index n denotes a variable under flexible prices. Furthermore, we can express the latter equation as:

$$\frac{\widetilde{X}_t}{\widetilde{Y}_t} = \left[1 - \alpha \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon - 1}\right)^{-1} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma - 1}\right)^{-1} \left(\frac{1-\xi_t^0}{(1+\varsigma_D)^{-1}}\right)\right]^{-1} \cdot \frac{X_t}{Y_t},$$

where $\widetilde{X}_t = X_t/X_t^n$ and $\widetilde{Y}_t = Y_t/Y_t^n$ represent the composite good and output gaps, respectively.

Equilibrium Conditions Summary We provide a summary of the equilibrium conditions under the assumption of zero trend inflation ($\Pi = 1$) and optimal government subsidies to firms, banks, and depositors ($\varsigma_F^* = \frac{1}{\epsilon-1}$; $\varsigma_B^* = \frac{1}{\sigma-1}$; $\varsigma_D^* = \frac{\xi^0}{1-\xi^0}$) to offset steady-state real distortions arising from monopolistic mark-ups and central bank intervention.

$$F_t = \left[1 - \alpha \cdot \frac{1-\xi_t^0}{1-\xi^0}\right]^{-1} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{-\left(\frac{\eta+1}{\eta+\alpha}\right)} \widetilde{X}_t^{(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta \cdot E_t \left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} F_{t+1}\right] , \quad \text{(A.20)}$$

$$H_t = \left[1 - \alpha \cdot \frac{1-\xi_t^0}{1-\xi^0}\right]^{-1} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{-1} + \theta\beta \cdot E_t \left[\Pi_{t+1}^{\epsilon-1} H_{t+1}\right] ,$$

$$\frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} , \quad \text{(A.21)}$$

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \cdot \Delta_{t-1} ,$$

$$\left[1 - \alpha \cdot \frac{1-\xi_t^0}{1-\xi^0}\right] \cdot \frac{\widetilde{X}_t}{\widetilde{Y}_t} = 1 - \alpha \left(\frac{1-\xi_t^0}{1-\xi^0}\right) \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{-\left(\frac{1-\alpha}{\eta+\alpha}\right)} \widetilde{X}_t^{(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)} \Delta_t , \quad \text{(A.22)}$$

$$\frac{1}{R_t^B} = \beta E_t \left[\frac{\widetilde{X}_t}{\widetilde{X}_{t+1}\Pi_{t+1}} \cdot \frac{X_t^n}{X_{t+1}^n}\right] , \quad \text{(A.23)}$$

$$R_t^B = R^B \; \Pi_t^{\gamma_\pi} \; \widetilde{Y}_t^{\gamma_y} \cdot exp\left(u_t^R\right) , \quad \text{(A.24)}$$

$$N_t = (1-\alpha)^{\frac{\eta}{\eta+1}} \left[1 - \alpha \cdot \frac{1-\xi_t^0}{1-\xi^0}\right]^{-\frac{\eta}{\eta+1}} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{-\left(\frac{\eta}{\eta+\alpha}\right)} \widetilde{X}_t^{(1-\alpha)\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}} , \quad \text{(A.25)}$$

$$X_t^n = \left[1 - \alpha \cdot \frac{1-\xi_t^0}{1-\xi^0}\right]^{\frac{1}{\eta+1}} (1-\alpha)^{-\frac{1}{\eta+1}} \left(\widetilde{R}_t^I\right)^{-\left(\frac{\alpha}{1-\alpha}\right)} \cdot exp\left(u_t^A\right) , \quad \text{(A.26)}$$

$$\xi_t^0 = \sum_{i=1}^N s_t^i \cdot \left[1 + e^{\kappa\cdot\varpi_1} \cdot \left(\frac{\Phi_t^i}{\Phi^i}\right)^{-\kappa\varpi_2}\right]^{-1} , \quad \text{(A.27)}$$

$$s_t^i = a_t^i \left(\frac{\widetilde{R}_t^{I,i}}{\widetilde{R}_t^I}\right)^{1-\sigma} ,$$

$$\widetilde{R}_t^I = \left[\sum_{i=1}^N a_t^i \cdot \left(\widetilde{R}_t^{I,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} , \quad \text{(A.28)}$$

$$\widetilde{R}_t^{I,i} = \Phi_t^i \cdot \left[1 + e^{-\kappa\cdot\varpi_1} \cdot \left(\frac{\Phi_t^i}{\Phi^i}\right)^{\kappa\varpi_2}\right]^{-1/\kappa} , \quad \text{(A.29)}$$

$$\Phi_t^i = \left[\sum_{n=1}^N \left(\left(1-\xi^0\right) \cdot d_t^{ni} T_t^n\right)^{-\kappa}\right]^{-\frac{1}{\kappa}} , \quad \text{(A.30)}$$

$$
\begin{aligned}
T_t^n &= T^n \cdot exp\left(u_t^{T,n}\right) \ , \\
d_t^{ni} &= \left(d^{ni}\right)^{\varrho} \cdot exp\left(u_t^{I,ni}\right) \ , \\
a_t^n &= \frac{a^n \cdot exp\left(u_t^{a,n}\right)}{\sum_{j=1}^{N} a^j \cdot exp\left(u_t^{a,j}\right)} \ ,
\end{aligned}
\tag{A.31}
$$

$$
\begin{aligned}
u_t^A &= \rho_A \cdot u_{t-1}^A + \varepsilon_t^A \ , \\
u_t^R &= \rho_R \cdot u_{t-1}^R + \varepsilon_t^R \ , \\
u_t^{T,n} &= \rho_I \cdot u_{t-1}^{T,n} + \varepsilon_t^{T,n} \ , \\
u_t^{a,n} &= \rho_I \cdot u_{t-1}^{a,n} + \varepsilon_t^{a,n} \ , \\
u_t^{I,ni} &= \rho_I \cdot u_t^{I,ni} + \varepsilon_t^{I,ni} \ .
\end{aligned}
\tag{A.32}
$$

Steady State Summary These are the steady-state values of the variables, assuming zero trend inflation and the implementation of optimal government subsidies. 2

$$
\begin{aligned}
F &= \frac{1}{(1-\alpha)(1-\theta\beta)} \ , \\
H &= \frac{1}{(1-\alpha)(1-\theta\beta)} \ , \\
\Delta &= 1 \ , \\
\widetilde{X} &= 1 \ , \\
\widetilde{Y} &= 1 \ , \\
\Phi^i &= \left[\sum_{n=1}^{N} \left(\left(1-\xi^0\right) \cdot d^{ni} T^n\right)^{-\kappa}\right]^{-\frac{1}{\kappa}} \ , \\
\widetilde{R}^I &= \left[\sum_{i=1}^{N} a^i \cdot \left(1-\xi^{0i}\right)^{\frac{1-\sigma}{\kappa}} \left(\Phi^i\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \ , \\
\frac{M}{P} &= \xi^0 \cdot \left(\frac{\alpha}{1-\alpha}\right) \left(\widetilde{R}^I\right)^{-\frac{1}{1-\alpha}} \ ,
\end{aligned}
$$

$$
\begin{aligned}
X^n &= \left(\widetilde{R}^I\right)^{-\left(\frac{\alpha}{1-\alpha}\right)} \ , \\
u^A &= 0 \ , \\
u^R &= 0 \ , \\
u^{T,n} &= 0 \ , \\
u^{a,n} &= 0 \ , \\
u^{I,ni} &= 0 \ , \\
N &= 1 \ , \\
R^B &= \beta^{-1} \ , \\
\xi^0 &= \left[1 + e^{\kappa \varpi_1}\right]^{-1} \ .
\end{aligned}
$$

We can further rewrite the steady-state aggregate interbank rate as follows:

$$
\widetilde{R}^I = \left(\lambda^{Own}\right)^{1/\kappa} \cdot \left(1 - \xi^0\right) \cdot \left[\sum_{i=1}^{N} a^i \cdot \left(1 - \xi^{0i}\right)^{\frac{1-\sigma}{\kappa}} \left(T^i\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \ ,
$$

where $\lambda^{Own} = \left[\sum_{i=1}^{N} s^i \cdot \left(\lambda^{ii}\right)^{\frac{\sigma-1}{\kappa}}\right]^{\frac{\kappa}{\sigma-1}}$ represents an index of the share of funds that banks obtain from their own depositors. The utility at the efficient steady-state can now be expressed as:

$$
U = \log\left(X^n\right) - \left(\frac{\eta}{\eta+1}\right) \int_0^1 N_\tau^{1+1/\eta} \, \mathrm{d}\tau \propto -\frac{1}{\kappa}\left(\frac{\alpha}{1-\alpha}\right) \cdot \log\left(\lambda^{Own}\right) - \left(\frac{\alpha}{1-\alpha}\right) \cdot \log\left(\widetilde{R}^{I,AU}\right) \ . \quad \text{(A.33)}
$$

First order log-linear equilibrium conditions We use hats to denote steady-state deviations and lowercase letters to refer to the logarithms of variables. Approximating equations (A.30) and (A.31), we obtain:

$$
\hat{\tilde{r}}_t^I = \sum_{i=1}^{N} s^i \cdot \left[\left(1 - \varpi_2 \xi^0\right) \cdot \sum_{n=1}^{N} \lambda^{ni} \cdot \left[\hat{u}_t^{T,n} + \hat{u}_t^{I,ni}\right] - \frac{\widehat{\log\left(a_t^i\right)}}{\sigma - 1}\right] \ . \quad \text{(A.34)}
$$

Combining (A.32)-(A.34), we obtain the following expression:

$$
\hat{\tilde{r}}_t^I = \rho_I \cdot \hat{\tilde{r}}_{t-1}^I + \left(1 - \varpi_2 \xi^0\right) \cdot \left[\hat{\varepsilon}_t^{T,r} + \hat{\varepsilon}_t^{I,r}\right] - \frac{\hat{\varepsilon}_t^a}{\sigma - 1} \ , \quad \text{(A.35)}
$$

$$
\text{where:} \quad \hat{\varepsilon}_t^a = \sum_{i=1}^{N} \left(s^i - a^i\right) \cdot \hat{\varepsilon}_t^{a,i} \ , \quad \hat{\varepsilon}_t^{T,r} = \sum_{i=1}^{N} \left[\frac{1 - \varpi_2 \xi^{0i}}{1 - \varpi_2 \xi^0}\right] s^i \sum_{n=1}^{N} \lambda^{ni} \cdot \hat{\varepsilon}_t^{T,n} \ ,
$$

$$
\hat{\varepsilon}_t^{I,r} = \sum_{i=1}^{N} \left[\frac{1 - \varpi_2 \xi^{0i}}{1 - \varpi_2 \xi^0}\right] s^i \sum_{n=1}^{N} \lambda^{ni} \cdot \hat{\varepsilon}_t^{I,ni} \ .
$$

Similarly, we obtain a log-linear approximation of equations (A.22) and (A.27) as:

$$\hat{\tilde{x}}_t - \hat{\tilde{y}}_t = -\alpha \left(\frac{\eta+1}{\eta}\right) \cdot \hat{\tilde{x}}_t \ , \tag{A.36}$$

$$\widehat{\log\left(\xi_t^0\right)} = \rho_I \cdot \widehat{\log\left(\xi_{t-1}^0\right)} + \kappa\varpi_2(1-\xi^0) \cdot \left[\hat{\varepsilon}_t^{T,\xi} + \hat{\varepsilon}_t^{I,\xi}\right] \ ,$$

$$\text{where:} \quad \hat{\varepsilon}_t^a = \sum_{i=1}^{N} \left(s^i - a^i\right) \cdot \hat{\varepsilon}_t^{a,i}; \quad \hat{\varepsilon}_t^{T,\xi} = \sum_{i=1}^{N} \left[\frac{1-\xi^{0i}}{1-\xi^0}\right] s^i \sum_{n=1}^{N} \lambda^{ni} \cdot \hat{\varepsilon}_t^{T,n} \ ,$$

$$\hat{\varepsilon}_t^{I,\xi} = \sum_{i=1}^{N} \left[\frac{1-\xi^{0i}}{1-\xi^0}\right] s^i \sum_{n=1}^{N} \lambda^{ni} \cdot \hat{\varepsilon}_t^{I,ni} \ .$$

The approximations of equations (A.20) - (A.21) can be expressed as follows:

$$\hat{f}_t = (1-\theta\beta)\left[-\left(\frac{\eta+1}{\eta+\alpha}\right)\left(\hat{\tilde{x}}_t - \hat{\tilde{y}}_t\right) + (1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)\hat{\tilde{x}}_t - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right) \cdot \log\left(\xi_t^0\right)\right] + \theta\beta\left[\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)E_t\left[\hat{\pi}_{t+1}\right] + E_t\left[\hat{f}_{t+1}\right]\right] \ , \tag{A.37}$$

$$\hat{h}_t = -(1-\theta\beta) \cdot \left[\left(\hat{\tilde{x}}_t - \hat{\tilde{y}}_t\right) + \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right) \cdot \log\left(\xi_t^0\right)\right] + \theta\beta\left[(\epsilon-1)E_t\left[\hat{\pi}_{t+1}\right] + E_t\left[\hat{h}_{t+1}\right]\right] \ ,$$

$$\hat{f}_t - \hat{h}_t = \left[1 + \epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\left(\frac{\theta}{1-\theta}\right)\hat{\pi}_t \ . \tag{A.38}$$

By combining equations (A.36)-(A.38), we derive the New-Keynesian Phillips Curve:

$$\hat{\pi}_t = \Omega \cdot \hat{\tilde{y}}_t + \beta \cdot E_t\left[\hat{\pi}_{t+1}\right] \ , \tag{A.39}$$

where $\Omega = (1-\alpha)\left(\frac{\eta+1}{\eta}\right)\left[1+\alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-1}\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]^{-1}\left(\frac{(1-\theta)(1-\theta\beta)}{\theta}\right)$. The log-linear approximations of equations (A.23) and (A.24) are as follows:

$$\hat{r}_t^B = \gamma_\pi \cdot \hat{\pi}_t + \gamma_y \cdot \hat{\tilde{y}}_t + \hat{u}_t^R \ , \tag{A.40}$$

$$-\hat{r}_t^B = \left[\hat{\tilde{x}}_t - E_t\left[\hat{\tilde{x}}_{t+1}\right]\right] - E_t\left[\hat{\pi}_{t+1}\right] + (1-\rho_A) \cdot \hat{u}_t^A - \left(\frac{\alpha}{1-\alpha}\right) \cdot \left[\hat{\tilde{r}}_t^I - E_t\left[\hat{\tilde{r}}_{t+1}^I\right]\right] + \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\eta+1}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left[\widehat{\log\left(\xi_t^0\right)} - E_t\left[\widehat{\log\left(\xi_{t+1}^0\right)}\right]\right] \ . \tag{A.41}$$

By combining equations (A.35), (A.36), (A.40), and (A.41), we derive the Dynamic IS Equation:

$$\hat{\tilde{y}}_t = -\left[1 + \alpha\left(\frac{\eta}{\eta+1}\right)\right] \cdot \left[\hat{r}_t^B - E_t\left[\hat{\pi}_{t+1}\right] - \hat{\iota}_t^n\right] + E_t\left[\hat{\tilde{y}}_{t+1}\right] \ , \tag{A.42}$$

where $\iota_t^n \equiv \left[(1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right) \cdot \tilde{r}_t^I - (1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\eta+1}\right)\left(\frac{\xi^0}{1-\xi^0}\right) \cdot \log\left(\xi_t^0\right) - (1-\rho_A) \cdot u_t^A\right]$ represents the natural interest rate under flexible prices.

First-Order Model Solution The first-order model solution consists of equations (A.39) and (A.42), which form a system of stochastic differential equations. These can be summarized in matrix form as follows:

$$A_0 \cdot \begin{bmatrix} \hat{\hat{y}}_t \\ \hat{\pi}_t \end{bmatrix} = B_0 \cdot E_t \begin{bmatrix} \hat{\hat{y}}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} + C_0 \cdot \begin{bmatrix} \hat{\hat{r}}^I_t \\ \widehat{\log\left(\xi^0_t\right)} \\ \hat{u}^A_t \\ \hat{u}^R_t \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \hat{\hat{r}}^I_t \\ \widehat{\log\left(\xi^0_t\right)} \\ \hat{u}^A_t \\ \hat{u}^R_t \end{bmatrix} = G_0 \cdot \begin{bmatrix} \hat{\hat{r}}^I_{t-1} \\ \widehat{\log\left(\xi^0_{t-1}\right)} \\ \hat{u}^A_{t-1} \\ \hat{u}^R_{t-1} \end{bmatrix} + G_1 \cdot \begin{bmatrix} \varepsilon^{T,r}_t \\ \varepsilon^{I,r}_t \\ \varepsilon^{T,\xi}_t \\ \varepsilon^{I,\xi}_t \\ \varepsilon^{a}_t \\ \varepsilon^{A}_t \\ \varepsilon^{R}_t \end{bmatrix},$$

where: $$A_0 = \begin{bmatrix} -\Omega & 1 \\ \left[1+\alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-1} + \gamma_y & \gamma_\pi \end{bmatrix}, \qquad B_0 = \begin{bmatrix} 0 & \beta \\ \left[1+\alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-1} & 1 \end{bmatrix},$$

$$C_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ (1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right) & -(1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\eta+1}\right)\left(\frac{\xi^0}{1-\xi^0}\right) & -(1-\rho_A) & -1 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} \left(1-\varpi_2\xi^0\right) & \left(1-\varpi_2\xi^0\right) & 0 & 0 & -\frac{1}{\sigma-1} & 0 & 0 \\ 0 & 0 & \kappa\varpi_2(1-\xi^0) & \kappa\varpi_2(1-\xi^0) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$G_0 = diag\left(\rho_I, \rho_I, \rho_A, \rho_R\right)$$

Solving forward the system of equations:

$$\begin{bmatrix} \hat{\hat{y}}_t \\ \hat{\pi}_t \end{bmatrix} = \underbrace{\left[\sum_{s=0}^{\infty} \left(A_0^{-1}B_0\right)^s \cdot \left(A_0^{-1}C_0\right) \cdot G_0^s\right]}_{\equiv\Psi} \cdot \begin{bmatrix} \hat{\hat{r}}^I_t \\ \widehat{\log\left(\xi^0_t\right)} \\ \hat{u}^A_t \\ \hat{u}^R_t \end{bmatrix}.$$

If the matrix $\left(I - G^T \otimes \left(A_0^{-1}B_0\right)\right)$ is invertible, a solution for Ψ can be obtained as follows:

$$vec(\Psi) = \left(I - G_0^T \otimes \left(A_0^{-1}B_0\right)\right)^{-1} \cdot vec(A_0^{-1}C_0)\,,$$

where $vec()$ denotes the matrix vectorization operation.

Appendix 2 Welfare

A second-order approximation of the representative household's utility around the efficient steady state with zero trend inflation, given by $\bar{\Pi} = 1$, is expressed as:

$$U_t - U = \hat{x}_t - \left[\hat{n}_t + \frac{1}{2}\left(1 + \frac{1}{\eta}\right)\hat{n}_t^2\right] + \text{h.o.t.} \; .$$

Employing the results from Appendix Appendix 3.A, the expected per-period utility can be represented as:

$$E\left[U_t - U\right] = E\left[\left(\frac{\alpha}{1-\alpha}\right)\left[\left(\frac{\xi^0}{1-\xi^0}\right)\cdot \widehat{\log\left(\xi_t^0\right)} - \hat{\hat{r}}_t^I\right] - \frac{\Lambda_r}{2}\cdot\left(\hat{\hat{r}}_t^I\right)^2 - \frac{\Lambda_\xi}{2}\cdot\widehat{\log\left(\xi_t^0\right)}^2 - \Lambda_{r\xi}\cdot\widehat{\log\left(\xi_t^0\right)}\cdot\hat{\hat{r}}_t^I\right] + \text{t.i.p.} + \text{h.o.t.}\,, \quad \text{(A.43)}$$

where Λ_r, Λ_ξ and $\Lambda_{r\xi}$ are constants derived in equation (A.48) that encapsulate the economy's sensitivity to credit spread shocks and central bank lending. Term t.i.p. stands for terms independent of policy, and term h.o.t. stands for higher-order terms contained in the approximation error of the equation. We define static gains from trade as the steady state utility change with respect to autarky, which is formally represented as:

$$\mathbb{J}^{ss} \equiv \frac{U - U^{AU}}{U_x X} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{\kappa}\cdot\log\left(\lambda^{Own}\right) \;,$$

where the last equality follows from equation (A.33). We define the stochastic version of the gains from trade as follows:

$$\begin{aligned}
\mathbb{J} \equiv E\left[\frac{U_t - U_t^{AU}}{U_x X}\right] = \mathbb{J}^{ss} &- E\left[\left(\frac{\alpha}{1-\alpha}\right)\cdot\left[\left(\frac{\xi^0}{1-\xi^0}\right)\cdot\left[\widehat{\log\left(\xi_t^0\right)} - \log\widehat{\left(\xi_t^{0,AU}\right)}\right] - \left(\hat{\hat{r}}_t^I - \hat{\hat{r}}_t^{I,AU}\right)\right]\right] \quad \text{(A.44)}\\
&- E\left[\frac{\Lambda_r}{2}\cdot\left[\left(\hat{\hat{r}}_t^I\right)^2 - \left(\hat{\hat{r}}_t^{I,AU}\right)^2\right] + \frac{\Lambda_\xi}{2}\cdot\left[\widehat{\log\left(\xi_t^0\right)}^2 - \log\widehat{\left(\xi_t^{0,AU}\right)}^2\right]\right]\\
&- E\left[\frac{\Lambda_{r\xi}}{2}\cdot\left[\hat{\hat{r}}_t^I\cdot\widehat{\log\left(\xi_t^0\right)} - \hat{\hat{r}}_t^{I,AU}\cdot\log\widehat{\left(\xi_t^{0,AU}\right)}\right]\right] \;,
\end{aligned}$$

where the equality arises from (A.43). As central bank policy parameters are components of these constants, the central bank can influence the economy's sensitivity to financial shocks by adjusting its intervention rules.

Appendix 3 Welfare Approximation, Technical Derivations

Appendix 3.A Second-Order Approximations

In this appendix, we compute the second-order log-linear approximations of variables utilized in determining the welfare approximation outlined in Appendix Appendix 2.

Price dispersion Following the standard New-Keynesian model (refer to, for instance, Galí (2015)), we derive an expression for the second-order approximation of price dispersion as a function of inflation:

$$\sum_{t=0}^{\infty}\beta^t \cdot E_0\left[\widehat{\log(\Delta_t)}\right] = \frac{\theta\epsilon}{2(1-\theta)(1-\theta\beta)\Theta}\sum_{t=0}^{\infty}\beta^t \cdot E_0\left[\hat{\pi}_t^2\right] \ , \quad \text{where: } \Theta = \left(\frac{\eta+1}{\eta+\alpha}\right)^{-1}\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]^{-1} \ . \quad \text{(A.45)}$$

Aggregate variables By employing implicit differentiation on equation (A.22), we obtain a second-order approximation of $\hat{\hat{x}}_t - \hat{\hat{y}}_t$ as a function of $\hat{\hat{x}}_t$ and $\widehat{\log(\Delta_t)}$

$$\begin{aligned}\hat{\hat{x}}_t - \hat{\hat{y}}_t = & -\alpha\left(\frac{\eta+1}{\eta}\right)\cdot\hat{\hat{x}}_t - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta}\right)\cdot\widehat{\log(\Delta_t)} - \frac{\alpha}{2}\left(\frac{\eta+1}{\eta}\right)^2\left(\frac{\eta+\alpha}{\eta}\right)\cdot\hat{\hat{x}}_t^2 \\ & + \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta+1}{\eta}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left(\frac{\eta+\alpha}{\eta}\right)\cdot\hat{\hat{x}}_t\cdot\widehat{\log(\xi_t^0)} \ .\end{aligned}$$

A second-order approximation of equations (A.25) and (A.26) is:

$$\begin{aligned}\hat{n}_t = & -\left(\frac{\eta}{\eta+\alpha}\right)(\hat{\hat{x}}_t - \hat{\hat{y}}_t) + (1-\alpha)\left(\frac{\eta}{\eta+\alpha}\right)\hat{\hat{x}}_t - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\widehat{\log(\xi_t^0)} \quad \text{(A.46)} \\ & + \left(\frac{\eta}{\eta+1}\right)\cdot\widehat{\log(\Delta_t)} - \frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left(\frac{1-\alpha-\xi^0}{(1-\alpha)(1-\xi^0)}\right)\cdot\widehat{\log(\xi_t^0)}^2 \ , \\ \hat{x}_t^n = & \hat{u}_t^A - \left(\frac{\alpha}{1-\alpha}\right)\cdot\hat{\hat{r}}_t^I + \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\eta+1}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\cdot\widehat{\log(\xi_t^0)} \quad \text{(A.47)} \\ & + \frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\eta+1}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left(\frac{1-\alpha-\xi^0}{(1-\alpha)(1-\xi^0)}\right)\cdot\widehat{\log(\xi_t^0)}^2 \ .\end{aligned}$$

Using equations (A.45), (A.46) and (A.47), we compute:

$$\sum_{t=0}^{\infty}\beta^t E\left[U_t - U\right] = \sum_{t=0}^{\infty}\beta^t E\Bigg[\left(\frac{\alpha}{1-\alpha}\right)\left[\left(\frac{\xi^0}{1-\xi^0}\right)\cdot\widehat{\log(\xi_t^0)} - \hat{\hat{r}}_t^I\right] + \frac{\Lambda_0}{2}\cdot\widehat{\log(\xi_t^0)}^2 + \Lambda_1\cdot\hat{\hat{y}}_t\cdot\widehat{\log(\xi_t^0)} - \frac{1}{2}\left[\Xi_1\cdot\hat{\pi}_t^2 + \Xi_2\cdot\hat{\hat{y}}_t^2\right]\Bigg] \ ,$$

$$\begin{aligned}\text{where:} \quad \Lambda_0 &= \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left[\frac{1-\alpha-\xi^0}{(1-\alpha)(1-\xi^0)} - \left(\frac{\eta}{\eta+1}\right)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\right] \ , \\ \Lambda_1 &= \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left[1+\left(\frac{\eta}{\eta+1}\right)\right]\left[1+\alpha\left(\frac{\eta}{\eta+1}\right)\right]^{-1} \ , \\ \Xi_1 &= \left(\frac{\varepsilon}{1-\alpha}\right)\cdot\left[1+\varepsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\cdot\left(\frac{\theta}{(1-\theta)(1-\theta\beta)}\right) \ , \\ \Xi_2 &= \left(\frac{\eta+1}{\eta}\right)\left[1+\alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-1} \ .\end{aligned}$$

By substituting $\hat{\hat{y}}_t$ and $\hat{\pi}_t$ using the first-order model solution, we obtain:

$$\sum_{t=0}^{\infty}\beta^t E\left[U_t - U\right] = \sum_{t=0}^{\infty}\beta^t E\Bigg[\left(\frac{\alpha}{1-\alpha}\right)\left[\left(\frac{\xi^0}{1-\xi^0}\right)\cdot\widehat{\log(\xi_t^0)} - \hat{\hat{r}}_t^I\right] - \frac{\Lambda_r}{2}\cdot\left(\hat{\hat{r}}_t^I\right)^2 + \frac{\Lambda_\xi}{2}\cdot\widehat{\log(\xi_t^0)}^2 + \Lambda_{r\xi}\cdot\widehat{\log(\xi_t^0)}\cdot\hat{\hat{r}}_t^I\Bigg] \ , \quad \text{(A.48)}$$

$$\begin{aligned}\text{where:} \quad \Lambda_r &= \Xi_1\cdot(\Psi_{21})^2 + \Xi_2\cdot(\Psi_{11})^2 \ , \quad \Lambda_\xi = \Lambda_0 + 2\Lambda_1\Psi_{12} - \left[\Xi_1\cdot(\Psi_{22})^2 + \Xi_2\cdot(\Psi_{12})^2\right] \ , \\ \Lambda_{r\xi} &= \Lambda_1\Psi_{11} - \left[\Xi_1\cdot\Psi_{21}\cdot\Psi_{22} + \Xi_2\cdot\Psi_{11}\cdot\Psi_{12}\right] \ .\end{aligned}$$

Banking variables A second-order approximation of equations (A.28) and (A.29) is:

$$\begin{aligned}\hat{\hat{r}}_t^I =& \sum_{i=1}^N s^i \left[\hat{\hat{r}}_t^{I,i} - \frac{\widehat{\log(a_t^i)}}{\sigma-1}\right] - \left(\frac{\sigma-1}{2}\right)\sum_{i=1}^N s^i \left[\left(\frac{1}{\sigma-1}\right)^2 \cdot \widehat{\log(a_t^i)}^2 + \left(\hat{\hat{r}}_t^{I,i}\right)^2 - 2\left(\frac{1}{\sigma-1}\right)\cdot \widehat{\log(a_t^i)} \cdot \hat{\hat{r}}_t^{I,i}\right] \quad \text{(A.49)}\\ &+ \left(\frac{\sigma-1}{2}\right)\sum_{i=1}^N\sum_{j=1}^N s^i s^j \left[\left(\frac{1}{\sigma-1}\right)^2 \cdot \widehat{\log(a_t^i)}\cdot\widehat{\log(a_t^j)} + \hat{\hat{r}}_t^{I,i}\hat{\hat{r}}_t^{I,j} - 2\left(\frac{1}{\sigma-1}\right)\cdot\widehat{\log(a_t^i)}\cdot\hat{\hat{r}}_t^{I,j}\right],\\ \hat{\hat{r}}_t^{I,i} =& \left(1-\varpi_2\xi^{0i}\right)\cdot\hat{\phi}_t^i - \frac{1}{2}\cdot\kappa\left(\varpi_2\right)^2\xi^{0i}\left(1-\xi^{0i}\right)\cdot\left(\hat{\phi}_t^i\right)^2 . \quad \text{(A.50)}\end{aligned}$$

By combining equations (A.49) and (A.50), we obtain:

$$\begin{aligned}\hat{\hat{r}}_t^I =& \sum_{i=1}^N s^i\cdot\left[\left(1-\varpi_2\xi^{0i}\right)\cdot\hat{\phi}_t^i - \frac{\widehat{\log\left(a_t^i\right)}}{\sigma-1}\right] - \left(\frac{\sigma-1}{2}\right)\sum_{i=1}^N s^i\left[\frac{\kappa}{\sigma-1}\left(\varpi_2\right)^2\xi^{0i}\left(1-\xi^{0i}\right) + \left(1-\varpi_2\xi^{0i}\right)^2\right]\cdot\left(\hat{\phi}_t^i\right)^2 \quad \text{(A.51)}\\ &- \left(\frac{\sigma-1}{2}\right)\sum_{i=1}^N s^i\cdot\left[\left(\frac{1}{\sigma-1}\right)^2\widehat{\log\left(a_t^i\right)}^2 - 2\left(1-\varpi_2\xi^{0i}\right)\cdot\hat{\phi}_t^i\cdot\frac{\widehat{\log\left(a_t^i\right)}}{\sigma-1}\right]\\ &+ \left(\frac{\sigma-1}{2}\right)\sum_{i=1}^N\sum_{j=1}^N s^i s^j\cdot\left[\left(1-\varpi_2\xi^{0i}\right)\left(1-\varpi_2\xi^{0j}\right)\cdot\hat{\phi}_t^i\cdot\hat{\phi}_t^j + \left(\frac{1}{\sigma-1}\right)^2\widehat{\log\left(a_t^i\right)}\cdot\widehat{\log\left(a_t^j\right)} - 2\left(1-\varpi_2\xi^{0i}\right)\cdot\phi_t^i\cdot\frac{\widehat{\log\left(a_t^j\right)}}{\sigma-1}\right].\end{aligned}$$

The second-order approximations for ξ_t^{0i}, ξ_t^0, and s_t^i are:

$$\begin{aligned}\widehat{\log\left(\xi_t^{0i}\right)} =& \kappa\varpi_2\left(1-\xi^{0i}\right)\cdot\hat{\phi}_t^i - \frac{1}{2}\cdot\kappa^2\left(\varpi_2\right)^2\cdot\xi^{0i}\left(1-\xi^{0i}\right)\cdot\left(\hat{\phi}_t^i\right)^2,\\ \widehat{\log\left(\xi_t^0\right)} =& \sum_{i=1}^N s^i\left(\frac{\xi^{0i}}{\xi^0}\right)\cdot\left[\widehat{\log\left(s_t^i\right)} + \widehat{\log\left(\xi_t^{0i}\right)}\right] + \frac{1}{2}\cdot\sum_{i=1}^N s^i\left(\frac{\xi^{0i}}{\xi^0}\right)\cdot\left[\widehat{\log\left(s_t^i\right)}^2 + \widehat{\log\left(\xi_t^{0i}\right)}^2 + 2\cdot\widehat{\log\left(s_t^i\right)}\cdot\widehat{\log\left(\xi_t^{0i}\right)}\right]\\ &- \frac{1}{2}\cdot\sum_{i=1}^N\sum_{j=1}^N s^i\left(\frac{\xi^{0i}}{\xi^0}\right)s^j\left(\frac{\xi^{0j}}{\xi^0}\right)\cdot\left[\widehat{\log\left(\xi_t^{0i}\right)}\cdot\widehat{\log\left(\xi_t^{0j}\right)} + \widehat{\log\left(s_t^i\right)}\cdot\widehat{\log\left(s_t^j\right)} + 2\cdot\widehat{\log\left(\xi_t^{0i}\right)}\cdot\widehat{\log\left(s_t^j\right)}\right],\\ \widehat{\log\left(s_t^i\right)} =& (\sigma-1)\left[\hat{\hat{r}}_t^I - \left[\hat{\hat{r}}_t^{I,i} - \frac{\widehat{\log\left(a_t^i\right)}}{\sigma-1}\right]\right]. \quad \text{(A.52)}\end{aligned}$$

Additionally, the second-order approximations of equations (A.30) and (A.31) are:

$$\begin{aligned}\hat{\phi}_t^i =& \sum_{n=1}^N\lambda^{ni}\cdot\left[\hat{u}_t^{T,n} + \hat{u}_t^{I,ni}\right] + \frac{\kappa}{2}\sum_{n=1}^N\sum_{j\neq n}\lambda^{ni}\lambda^{ji}\cdot\left[\hat{u}_t^{T,n}\cdot\hat{u}_t^{T,j} + \hat{u}_t^{I,ni}\cdot\hat{u}_t^{I,ji} + 2\cdot\hat{u}_t^{T,j}\cdot\hat{u}_t^{I,ni}\right] \quad \text{(A.53)}\\ &- \frac{\kappa}{2}\sum_{n=1}^N\lambda^{ni}(1-\lambda^{ni})\cdot\left[\left(\hat{u}_t^{T,n}\right)^2 + \left(\hat{u}_t^{I,ni}\right)^2 + 2\cdot\hat{u}_t^{T,n}\cdot\hat{u}_i^{I,ni}\right],\\ \widehat{\log\left(a_t^n\right)} =& \hat{u}_t^{a,n} - \sum_{i=1}^N a^i\hat{u}_t^{a,i} - \frac{1}{2}\sum_{i=1}^N a^i\left(1-a^i\right)\cdot\left(\hat{u}_t^{a,i}\right)^2 + \frac{1}{2}\sum_{i=1}^N\sum_{n\neq i}a^i a^n\cdot\hat{u}_t^{a,i}\cdot\hat{u}_t^{a,n}.\end{aligned}$$

It follows from the previous expressions that:

$$\hat{\phi}_t^i\cdot\hat{\phi}_t^n = \sum_{j=1}^N\sum_{k=1}^N\lambda^{ji}\lambda^{kn}\cdot\left[\hat{u}_t^{T,j}\cdot\hat{u}_t^{T,k} + \hat{u}^{I,ji}\cdot\hat{u}^{I,kn} + \hat{u}_t^{T,j}\cdot\hat{u}^{I,kn} + \hat{u}_t^{T,k}\cdot\hat{u}^{I,ji}\right], \quad \text{(A.54)}$$

$$\widehat{\log\left(a_t^i\right)}\cdot\widehat{\log\left(a_t^n\right)} = \hat{u}_t^{a,i}\cdot\hat{u}_t^{a,n} - \sum_{j=1}^N a^j\cdot\hat{u}_t^{a,n}\cdot\hat{u}_t^{a,j} - \sum_{k=1}^N a^k\cdot\hat{u}_t^{a,i}\cdot\hat{u}_t^{a,k} + \sum_{j=1}^N\sum_{k=1}^N a^j a^k\cdot\hat{u}_t^{a,j}\cdot\hat{u}_t^{a,k}. \quad \text{(A.55)}$$

By combining equations (A.48) and (A.51)-(A.52), we derive an expression for the household's utility:

$$
\begin{aligned}
E\left[U_t - U\right] = & - \sum_{i=1}^{N} s^i \cdot \aleph_{0,i} \cdot E\left[\hat{\phi}_t^i\right] + \sum_{i=1}^{N} s^i \cdot \aleph_{1,i} \cdot E\left[\widehat{\log(a_t^i)}\right] + \frac{1}{2} \sum_{i=1}^{N} s^i \cdot \aleph_{2,i} \cdot E\left[\left(\hat{\phi}_t^i\right)^2\right] - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} s^i s^j \cdot \aleph_{3,ij} \cdot E\left[\hat{\phi}_t^i \cdot \hat{\phi}_t^j\right] \\
& + \frac{1}{2} \sum_{i=1}^{N} s^i \cdot \aleph_4 \cdot E\left[\widehat{\log(a_t^i)}^2\right] - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} s^i s^j \cdot \aleph_{5,ij} \cdot E\left[\widehat{\log(a_t^i)} \cdot \widehat{\log(a_t^j)}\right] \\
& - \sum_{i=1}^{N} s^i \cdot \aleph_{6,i} \cdot E\left[\hat{\phi}_t^i \cdot \widehat{\log(a_t^i)}\right] + \sum_{i=1}^{N} \sum_{j=1}^{N} s^i s^j \cdot \aleph_{7,ij} \cdot E\left[\hat{\phi}_t^i \cdot \widehat{\log(a_t^j)}\right] ,
\end{aligned}
$$

where:

$$
\aleph_{0,i} = \left(\frac{\alpha}{1-\alpha}\right)\left[(1-\varpi_2\xi^{0i}) - \left(\frac{(\sigma-1)\xi^0}{1-\xi^0}\right)\left[\left(1-\frac{\xi^{0i}}{\xi^0}\right)(1-\varpi_2\xi^{0i}) + \left(\frac{\xi^{0i}}{\xi^0}\right)\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^{0i})\right]\right] , \qquad \aleph_{1,i} = \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{1}{\sigma-1} - \left(1-\frac{\xi^{0i}}{\xi^0}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\right] ,
$$

$$
\begin{aligned}
\aleph_{2,i} = & \left(\frac{\alpha(\sigma-1)^2}{1-\alpha}\right)\left[\frac{\kappa}{(\sigma-1)^2}(\varpi_2)^2\xi^{0i}(1-\xi^{0i}) + \frac{(1-\varpi_2\xi^{0i})}{\sigma-1}\right] \\
& - \left(\frac{\alpha(\sigma-1)^2}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left[\left(\frac{\xi^{0i}}{\xi^0}\right)\left[2\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^{0i})(1-\varpi_2\xi^{0i}) - \frac{\kappa^2(\varpi_2)^2}{(\sigma-1)^2}(1-\xi^{0i})(1-2\xi^{0i})\right] + \left(1-\frac{\xi^{0i}}{\xi^0}\right)\frac{\kappa(\varpi_2)^2}{\sigma-1}\xi^{0i}(1-\xi^{0i})\right] ,
\end{aligned}
$$

$$
\begin{aligned}
\aleph_{3,ij} = & \left[\frac{\alpha(\sigma-1)}{1-\alpha} + \Lambda_r\right](1-\varpi_2\xi^{0i})(1-\varpi_2\xi^{0j}) - \left(\frac{\alpha(\sigma-1)^2}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left[1-\left(\frac{\xi^{0i}}{\xi^0}\right)\left(\frac{\xi^{0j}}{\xi^0}\right)\right](1-\varpi_2\xi^{0i})(1-\varpi_2\xi^{0j}) \\
& - \left(\frac{\alpha(\sigma-1)^2}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left(\frac{\xi^{0i}}{\xi^0}\right)\left(\frac{\xi^{0j}}{\xi^0}\right)\left[2\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^{0i})(1-\varpi_2\xi^{0j}) - \frac{\kappa^2(\varpi_2)^2}{(\sigma-1)^2}(1-\xi^{0i})(1-\xi^{0j})\right] \\
& - \Lambda_\xi(\sigma-1)^2\left[\left(1-\frac{\xi^{0i}}{\xi^0}\right)(1-\varpi_2\xi^{0i}) + \left(\frac{\xi^{0i}}{\xi^0}\right)\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^{0i})\right]\left[\left(1-\frac{\xi^{0j}}{\xi^0}\right)(1-\varpi_2\xi^{0j}) + \left(\frac{\xi^{0j}}{\xi^0}\right)\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^{0j})\right] \\
& - 2\Lambda_{r\xi}(\sigma-1)\left[\left(1-\frac{\xi^{0i}}{\xi^0}\right)(1-\varpi_2\xi^{0i}) + \left(\frac{\xi^{0i}}{\xi^0}\right)\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^{0i})\right]\left(1-\varpi_2\xi^{0j}\right) ,
\end{aligned}
$$

$$
\aleph_4 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\sigma-1}\right) , \qquad \aleph_{5,ij} = \left(\frac{1}{\sigma-1}\right)\left[\left(\frac{\alpha}{1-\alpha}\right) + \frac{\Lambda_r}{\sigma-1}\right] - \Lambda_\xi\left(1-\frac{\xi^{0i}}{\xi^0}\right)\left(1-\frac{\xi^{0j}}{\xi^0}\right) - 2\Lambda_{r\xi}\left(1-\frac{\xi^{0i}}{\xi^0}\right)\left(\frac{1}{\sigma-1}\right) - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left[1-\left(\frac{\xi^{0i}}{\xi^0}\right)\left(\xi^0 \ldots\right.\right.
$$

$$
\aleph_{6,i} = \left(\frac{\alpha}{1-\alpha}\right)\left[(1-\varpi_2\xi^{0i}) - \left(\frac{\xi^0}{1-\xi^0}\right)\left(\frac{\xi^{0i}}{\xi^0}\right)\kappa\varpi_2(1-\xi^{0i})\right] ,
$$

$$
\begin{aligned}
\aleph_{7,ij} = & \left(\frac{\alpha}{1-\alpha}\right)(\sigma-1)\left[\left(\frac{\xi^0}{1-\xi^0}\right)\left[\left(1-\frac{\xi^{0i}}{\xi^0}\right)(1-\xi^{0i})\frac{\kappa\varpi_2}{\sigma-1} - \left[1-\left(\frac{\xi^{0i}}{\xi^0}\right)\left(\frac{\xi^{0j}}{\xi^0}\right)\right](1-\varpi_2\xi^{0i})\right] + \left(\frac{1-\varpi_2\xi^{0i}}{\sigma-1}\right)\right] \\
& + \Lambda_r\left(\frac{1-\varpi_2\xi^{0i}}{\sigma-1}\right) - \Lambda_\xi(\sigma-1)\left[\left(1-\frac{\xi^{0i}}{\xi^0}\right)(1-\varpi_2\xi^{0i}) + \left(\frac{\xi^{0i}}{\xi^0}\right)\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^{0i})\right]\left(1-\frac{\xi^{0j}}{\xi^0}\right) - \Lambda_{r\xi}\left[\left(2-\frac{\xi^{0i}}{\xi^0}-\frac{\xi^{0j}}{\xi^0}\right)(1-\varpi_2\xi^{0i}) + \left(\frac{\xi^{0i}}{\xi^0}\right)\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^{0i} \ldots\right.
\end{aligned}
$$

In the baseline case, where all banks have equal access to the central bank, $\xi^{0i} = \xi^0, \ \forall j$, the above expression simplifies as follows:

$$
\begin{aligned}
E\left[U_t - U\right] = & - \aleph_0 \cdot \sum_{i=1}^{N} s^i \cdot E\left[\hat{\phi}_t^i\right] + \aleph_1 \cdot \sum_{i=1}^{N} s^i \cdot E\left[\widehat{\log(a_t^i)}\right] + \frac{\aleph_2}{2} \cdot \sum_{i=1}^{N} s^i \cdot E\left[\left(\hat{\phi}_t^i\right)^2\right] - \frac{\aleph_3}{2} \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} s^i s^j \cdot E\left[\hat{\phi}_t^i \cdot \hat{\phi}_t^j\right] + \frac{\aleph_4}{2} \cdot \sum_{i=1}^{N} s^i E\left[\widehat{\log(a_t^i)}^2\right] \\
& - \frac{\aleph_5}{2} \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} s^i s^j \cdot E\left[\widehat{\log(a_t^i)} \cdot \widehat{\log(a_t^j)}\right] - \aleph_6 \cdot \sum_{i=1}^{N} s^i \cdot E\left[\hat{\phi}_t^i \cdot \widehat{\log(a_t^i)}\right] + \aleph_7 \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} s^i s^j \cdot E\left[\hat{\phi}_t^i \cdot \widehat{\log(a_t^j)}\right] ,
\end{aligned}
\tag{A.56}
$$

where:

$$
\aleph_0 = \left(\frac{\alpha}{1-\alpha}\right)\left[(1-\varpi_2\xi^0) - \left(\frac{\xi^0}{1-\xi^0}\right)\kappa\varpi_2(1-\xi^0)\right] , \qquad \aleph_1 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\sigma-1}\right) ,
$$

$$
\aleph_2 = \left(\frac{\alpha(\sigma-1)^2}{1-\alpha}\right)\left[\frac{\kappa(\varpi_2)^2}{(\sigma-1)^2}\xi^0(1-\xi^0) + \frac{1-\varpi_2\xi^0}{\sigma-1}\right] - \left(\frac{\alpha(\sigma-1)^2}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left[2\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^0)(1-\varpi_2\xi^0) - \frac{\kappa^2(\varpi_2)^2}{(\sigma-1)^2}(1-\xi^0)(1-2\xi^0)\right] ,
$$

$$
\aleph_3 = \left[\left(\frac{\alpha(\sigma-1)}{1-\alpha}\right) + \Lambda_r\right](1-\varpi_2\xi^0)^2 - \Lambda_\xi\kappa^2(\varpi_2)^2(1-\xi^0)^2 - 2\Lambda_{r\xi}\kappa\varpi_2(1-\xi^0)(1-\varpi_2\xi^o) - \left(\frac{\alpha(\sigma-1)^2}{1-\alpha}\right)\left(\frac{\xi^0}{1-\xi^0}\right)\left[2\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^0)(1-\varpi_2\xi^0) - \frac{\kappa^2(\varpi_2)^2}{(\sigma-1)^2}(1 \ldots\right.
$$

$$
\aleph_4 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\sigma-1}\right) , \qquad \aleph_5 = \left(\frac{1}{\sigma-1}\right)\left[\left(\frac{\alpha}{1-\alpha}\right) + \frac{\Lambda_r}{\sigma-1}\right] ,
$$

$$
\aleph_6 = \left(\frac{\alpha}{1-\alpha}\right)\left[(1-\varpi_2\xi^0) - \left(\frac{\xi^0}{1-\xi^0}\right)\kappa\varpi_2(1-\xi^0)\right] , \qquad \aleph_7 = \left[\left(\frac{\alpha}{1-\alpha}\right) + \left(\frac{\Lambda_r}{\sigma-1}\right)\right](1-\varpi_2\xi^0) - \Lambda_{r\xi}\frac{\kappa\varpi_2}{\sigma-1}(1-\xi^0) .
$$

Appendix 3.B Welfare Derivations

Additional Assumptions To derive a tractable welfare expression in equation (A.44), we impose the following additional assumptions. 2 *1. CES firm weights:*

$$E\left[u_t^{a,i}\cdot u_t^{a,n}\right]=\begin{cases}\sigma_a^2 & \text{if } n=i\ ,\\ \zeta_a\cdot\sigma_a^2 & \text{otherwise}\ .\end{cases}$$

2. Depositor Preferences:

$$E\left[u_t^{T,i}\cdot u_t^{T,i}\right]=\begin{cases}\sigma_T^2 & \text{if } n=i\ ,\\ \zeta_T\cdot\sigma_T^2 & \text{otherwise}\ .\end{cases}$$

3. Bilateral Transaction Costs:

$$E\left[u_t^{I,ji}\cdot u_t^{I,kn}\right]=\begin{cases}0 & \text{if } j=i \text{ or } k=n\ ,\\ \sigma_I^2 & \text{if } k=j,\ n=i\ ,\\ \zeta_{I,B}\cdot\sigma_I^2 & \text{if } k\neq j,\ n=i\ ,\\ \zeta_{I,L}\cdot\sigma_I^2 & \text{if } k=j,\ n\neq i\ ,\\ \zeta_{I,X}\cdot\sigma_I^2 & \text{otherwise}\ .\end{cases}$$

4. Zero Cross-Correlation:

$$E\left[u_t^{I,ji}\cdot u_t^{a,k}\right]=E\left[u_t^{I,ji}\cdot u_t^{T,k}\right]=E\left[u_t^{a,j}\cdot u_t^{T,k}\right]=0\ ,\quad \forall j,i,k\ .$$

Welfare Expectations The expectations of (A.53)-(A.55) are

$$E\left[\hat{\phi}_t^i\right]=-\frac{\kappa}{2}\left[\sigma_T^2(1-\zeta_T)\cdot\left[1-H^{I,i}\right]+\sigma_I^2\cdot\left[(1-\lambda^{ii})-\left[H^{I,i}-(\lambda^{ii})^2\right]\right]-\zeta_{I,B}\cdot\sigma_I^2\cdot\left[H^{O,i}-H^{I,i}\right]\right]\ , \tag{A.57}$$

$$E\left[\widehat{\log\left(a_t^i\right)}\right]=-\frac{\sigma_a^2}{2}\left(1-\zeta_a\right)\cdot\left[1-H^a\right]\ ,$$

$$E\left[\hat{\phi}_t^i\cdot\hat{\phi}_t^n\right]=\begin{cases}\sigma_T^2(1-\zeta_T)\cdot\left[\frac{\zeta_T}{1-\zeta_T}+H^{I,i}\right]+\sigma_I^2\left[H^{I,i}-\left(\lambda^{ii}\right)^2\right]+\zeta_{I,B}\cdot\sigma_I^2\cdot\left[H^{O,i}-H^{I,i}\right]\ , & \text{if } n=i\ ,\\ \sigma_T^2(1-\zeta_T)\cdot\left[\frac{\zeta_T}{1-\zeta_T}+\sum_{j=1}^N\lambda^{ji}\lambda^{jn}\right]+\zeta_{I,L}\sigma_I^2\cdot\sum_{j\neq\{i,n\}}\lambda^{ji}\lambda^{jn}+\zeta_{I,X}\sigma_I^2\cdot\left[\left(1-\lambda^{ii}\right)(1-\lambda^{nn})-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]\ , & \text{otherwise}\ ,\end{cases}$$

$$E\left[\widehat{\log\left(a_t^i\right)}\cdot\widehat{\log\left(a_t^n\right)}\right]=\begin{cases}\sigma_a^2\cdot(1-\zeta_a)\cdot\left[1+H^a-2\cdot a^i\right]\ , & \text{if } n=i\ ,\\ \sigma_a^2(1-\zeta_a)\cdot\left[H_a-\left[a^i+a^n\right]\right]\ , & \text{otherwise}\ ,\end{cases} \tag{A.58}$$

where we define $H^a=\sum_{n=1}^N\left(a^n\right)^2$ as the Herfindahl index of concentration of firm loan demand, $H^{I,i}=\sum_{j=1}^N\left(\lambda^{ji}\right)^2$ as the Herfindahl index of concentration of bank i funding sources, and $H^{O,i}=\left(\lambda^{ii}\right)^2+\left(1-\lambda^{ii}\right)^2$ as the Herfindahl index of concentration of bank's i own vs. outside funding. Plugging (A.57)-(A.58) into (A.56), we obtain:

$$
\begin{aligned}
E\left[U_t - U\right] = & \frac{\sigma_a^2}{2}(1-\zeta_a)\left[\aleph_5 \cdot \left[1 - H^F\right] - \left[\aleph_1 + \aleph_4 - \aleph_5\right] \cdot \left[1 - H^a\right] - 2(\aleph_5 - \aleph_4) \cdot \left[1 - \sum_{i=1}^{N} s^i a^i\right]\right] \\
& + \frac{\sigma_T^2}{2}(1-\zeta_T) \cdot \left[\frac{\aleph_2 - \zeta_T \aleph_3}{1-\zeta_T} - \aleph_3 \cdot H^F + \left[(\aleph_0 \kappa - \aleph_2) + \aleph_3 \cdot H^F\right] \cdot \left[1 - H^I\right] - \aleph_3 \cdot \sum_{i=1}^{N} \sum_{n \neq i} \sum_{j=1}^{N} s^i s^n \lambda^{ji} \lambda^{jn}\right] \\
& + \frac{\sigma_I^2}{2} \cdot \left[\aleph_0 \kappa \cdot \left(1 - \lambda^{Avg}\right) - \left[(\aleph_0 \kappa - \aleph_2) + \aleph_3 \cdot H^F\right] \cdot \beth^I \cdot H^I\right] \\
& - \frac{\sigma_I^2}{2} \cdot \zeta_{I,B} \cdot \left[(\aleph_0 \kappa - \aleph_2) + \aleph_3 \cdot H^F\right] \cdot \left[H^O - H^I\right] \\
& - \frac{\sigma_I^2}{2} \cdot \zeta_{I,L} \cdot \aleph_3 \cdot \sum_{i=1}^{N} \sum_{n \neq i} \sum_{j \neq \{i,n\}} s^i s^n \lambda^{ji} \lambda^{jn} \\
& - \frac{\sigma_I^2}{2} \cdot \zeta_{I,X} \cdot \aleph_3 \cdot \sum_{i=1}^{N} \sum_{n \neq i} s^i s^n \left[(1-\lambda^{ii})(1-\lambda^{nn}) - \sum_{j \neq i} \lambda^{ji} \lambda^{jn}\right] ,
\end{aligned}
$$

$$
\text{where:} \quad \omega^i = \frac{(\aleph_0 \kappa - \aleph_2) \cdot s^i + \aleph_3 \cdot (s^i)^2}{(\aleph_0 \kappa - \aleph_2) + \aleph_3 \cdot H^F} , \quad H^I = \sum_{i=1}^{N} \omega^i \cdot H^{I,i} , \quad H^O = \sum_{i=1}^{N} \omega^i \cdot H^{O,i} ,
$$

$$
\beth^I = \sum_{i=1}^{N} \omega^i \cdot \left(\frac{H^{I,i}}{H^I}\right) \left(\frac{H^{I,i} - \left(\lambda^{ii}\right)^2}{H^{I,i}}\right) , \quad \lambda^{Avg} = \sum_{i=1}^{N} s^i \cdot \lambda^{ii} .
$$

The variable $\beth^I$ represents the share of a bank's funding concentration attributed to outside sources. Finally, we derive an expression for the dynamic gains from trade as follows:

$$
\mathbb{J} = \mathbb{J}^{ss} + \frac{1}{2}\left[\sigma_a^2 \cdot \mathfrak{J}^a + \sigma_T^2 \cdot \mathfrak{J}^T + \sigma_I^2 \cdot \mathfrak{J}^I\right] ,
$$

$$
\begin{aligned}
\text{where:} \quad \mathfrak{J}^a = & (1-\zeta_a)\left[\aleph_5 \cdot \left[H^{F,AU} - H^F\right] - 2(\aleph_5 - \aleph_4) \cdot \sum_{i=1}^{N}\left(s^{i,AU} - s^i\right) a^i\right] , \\
\mathfrak{J}^T = & (1-\zeta_T) \cdot \left[\aleph_3 \cdot \left[H^{F,AU} - H^F\right] + \left[(\aleph_0 \kappa - \aleph_2) + \aleph_3 \cdot H^F\right] \cdot \left[1 - H^I\right] - \aleph_3 \cdot \sum_{i=1}^{N} \sum_{n \neq i} \sum_{j=1}^{N} s^i s^n \lambda^{ji} \lambda^{jn}\right] , \\
\mathfrak{J}^I = & \aleph_0 \kappa \cdot \left(1 - \lambda^{Avg}\right) - \left[(\aleph_0 \kappa - \aleph_2) + \aleph_3 \cdot H^F\right] \cdot \beth^I \cdot H^I \\
& - \zeta_{I,B} \cdot \left[(\aleph_0 \kappa - \aleph_2) + \aleph_3 \cdot H^F\right] \cdot \left[H^O - H^I\right] \\
& - \zeta_{I,L} \cdot \aleph_3 \cdot \sum_{i=1}^{N} \sum_{n \neq i} \sum_{j \neq \{i,n\}} s^i s^n \lambda^{ji} \lambda^{jn} \\
& - \zeta_{I,X} \cdot \aleph_3 \cdot \sum_{i=1}^{N} \sum_{n \neq i} s^i s^n \left[(1-\lambda^{ii})(1-\lambda^{nn}) - \sum_{j \neq i} \lambda^{ji} \lambda^{jn}\right] .
\end{aligned}
$$

Under the assumptions of no central bank direct lending, $\left\{\xi^0 = 0,\ \varpi_2 = 0\right\}$, and no correlation across distinct interbank transaction costs, $\zeta_{I,B} = \zeta_{I,L} = \zeta_{I,X} = 0$, the expression for multiplier

$\mathfrak{I}^I$ is proportional to:

$$\mathfrak{I}^I \propto \sum_{n=1}^{N} s^n \cdot [1 - \lambda^{nn}] - \left[\Theta_0 + \Theta_1 \cdot H^F\right] \cdot \sum_{n=1}^{N} \omega^n \cdot \left[H^{I,n} - (\lambda^{nn})^2\right] \ ,$$

where $\Theta_0 = 1 - \left(\frac{\sigma-1}{\kappa}\right)$ and $\Theta_1 = \left(\frac{\sigma-1}{\kappa}\right) + \left(\frac{1-\alpha}{\alpha}\right) \cdot \frac{\Lambda_r}{\kappa}$.