A Unified Theory of the Term-Structure and Monetary Stabilization*

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Abstract

We develop a New-Keynesian framework that incorporates the term-structure of financial markets and emphasizes the active role of the government and central bank's balance sheet size and composition. We demonstrate that the *financial market segmentation* and the household's *endogenous portfolio reallocation* are crucial features for accurately understanding the effects of Large-Scale Asset Purchase (LSAP) programs. Our micro-foundation based on imperfect information about expected discounted asset returns readily accommodates varying degrees of market segmentation across asset classes and maturities, based on estimatable asset demand elasticities. The central bank's bond purchases across maturities serve as a major determinant of the level and slope of the term-structure, and yield-curve-control (YCC) policies that actively manipulate long-term yields are highly effective in terms of stabilization during both normal times and at the ZLB. However, YCC policies also increase the durations of ZLB episodes, consequently placing the central bank in a position where the short-term rate becomes a less useful policy tool.

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1 Introduction

In recent decades, unconventional monetary policies[1] once considered unorthodox, have become mainstream for central banks, particularly following the 2007-2008 Global Financial Crisis and the subsequent Great Recession. Faced with the constraints of the zero-lower bound (ZLB) on short-term policy rates, policymakers turned to innovative strategies to lower long-maturity rates, aiming to stimulate aggregate demand and alleviate recessionary pressures. In pursuit of these objectives, central banks dramatically expanded their balance sheets, while governments worldwide increased their debt issuance to finance higher spending. The Covid-19 pandemic further intensified this extraordinary economic landscape, prompting the Federal Reserve to embark on yet another wave of unconventional interventions as the policy rate once again reached the ZLB.[2]

The standard log-linearized New-Keynesian framework incorporates a single policy rate, neglecting the term structure of interest rates and the heterogeneous returns across multiple assets. Integrating these omitted elements is not a straightforward task, as equilibrium typically equalizes expected returns across assets and maturities in these linearized models.[3] Consequently, any additional assets would become a fully dependent function of the policy rate, rendering them superfluous for the examination of monetary policy.

In this paper, we develop a tractable New-Keynesian framework that incorporates the endogenous term structure of interest rates in bond and private capital markets, allowing us to examine the effects of alternative monetary (i.e., conventional and unconventional) and fiscal policies. Informed by previous theoretical and empirical studies that emphasize *market segmentation* across bonds of varying maturities as a crucial aspect in explaining the effectiveness of quantitative easing programs, we offer a novel micro-foundation that enables the integration of (i) financial market segmentation, (ii) the household's endogenous portfolio balancing across different asset classes and maturities, and (iii) the real effects of government's and central bank's balance sheet sizes and compositions: all essential com-

[1]Examples include Quantitative Easing (QE) programs, large scale asset purchases (LSAPs) programs, and Operation Twist (OT).

[2]In March 2020, the Federal Reserve (Fed) lowered its policy rate tool to a range from 0% to 0.25%. The Fed committed to keeping interest rates low until the economy achieved full employment and maintained 2% inflation consistently. Concurrently, the unprecedented CARES Act injected nearly $500 billion in support of the Fed.

[3]This outcome arises from the usual absence of price of risk under the first-order log-linear approximation techniques, leading to the well-known *expectation hypothesis*, which holds true in most log-linearized New-Keynesian models. According to this hypothesis, long-term bond returns are simply the average of expected future short-term rates.

ponents for understanding the transmission channels of unconventional monetary policies.[4]

In the context of financial market segmentation, our framework posits that the total volume and maturity structure of the government's bond issuance influences the equilibrium levels of interest rates and the slope of the yield curve. In addition, the central bank's relative bond purchases across different maturities exhibit a negative relationship with bond yields. These outcomes align with the findings of Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2014), which underscore the short- and long-term significance of both relative asset demand and supply across maturities in shaping the yield curve.

We also investigate the cyclical properties of various monetary interventions in the form of simple policy rules. By explicitly integrating the government and central bank's balance sheets, our model readily facilitates the study of policies aimed at controlling yields, bond supplies, or a combination thereof at different maturities. We initially concentrate on the implementation of a conventional policy rule for the short-term rate, examining its impact on the entire yield curve and the economy. Subsequently, we devise a more comprehensive *yield-curve-control* (YCC) policy, wherein the central bank directly manipulates the entire yield curve of the bond market. Our framework uncovers interesting phenomena and differences across policies, particularly relevant when the economy encounters a ZLB episode (and thus, controlling the short-term rate is restricted). For instance, when the central bank adheres to a conventional monetary policy for short-term rates, a decrease in the government's risk-free bond supply proves recessionary at the ZLB, as posited by the literature on safe-asset shortage problems (see, for example, Caballero and Farhi (2017) and Caballero et al. (2021)). In contrast, under the YCC policy, the central bank rapidly shifts the entire yield curve downward, reducing the effective savings rate for households and stimulating aggregate demand to avert the economy's collapse.[5] We discover that YCC, in general, constitutes a more potent policy for economic stabilization and improves household welfare compared to a conventional short-term rate policy.

However, the YCC policy exhibits intriguing side effects, including more protracted

[4]For instance, the households' endogenous portfolio decisions play a key role in our model: a relative decline in the short-term rate induces household to reallocate their savings to other assets and/or longer-maturity bonds, diminishing the marginal effects of further policy rate changes on the household's intertemporal consumption decision and generating spillover effects relevant to the determination of other rates.

[5]Even under the conventional policy, a declining short-term rate lowers long-term bond yields due to the endogenous portfolio reallocation of the household, thereby diminishing the effective savings rate. However, this channel proves insufficient for boosting aggregate demand, especially when the economy reaches the ZLB constraint and the conventional policy becomes inoperative.

ZLB episodes. Actively easing long-maturity yields exerts supplementary downward pressure on short-term bond returns, resulting from the household's endogenous portfolio reallocation. Declining long-term rates prompt the household to withdraw its wealth from long-term bonds and reallocate investments into: (i) short-maturity bonds, which further depresses short-term yields, and (ii) private loan markets, thereby reducing firms' borrowing costs and subsequently consumption prices due to lower production costs.[6] When the ZLB is binding, the YCC policy disproportionately acts via the manipulation of long-term bond yields, which imposes additional downward pressure on short-term rates and postpones an exit from the ZLB. Consequently, the household's endogenous portfolio reallocation generates a feedback loop between ZLB duration and the necessity for YCC policies: YCC amplifies ZLB duration, while the economy increasingly depends on YCC's stabilization capacity during ZLB episodes. To the best of our knowledge, this outcome represents a novel contribution to the literature.[7]

We put forth an original microfoundation for financial market segmentation, based on imperfect information regarding asset returns. We assume a household subdivided into a continuum of families and family members, each possessing distinct and imperfect information sets about future asset returns, while perfect consumption insurance exists within the household. Subsequently, and unable to extract a common signal from the diverse information sets, the household uniformly apportions aggregate savings among its members, allowing them to allocate their share to the assets they perceive as most profitable. This investment strategy effectively culminates in market segmentation, where cross-sectional dispersion in each individual's expectation of asset returns dictates the degrees of market segmentation associated with each asset class. To simplify the aggregation problem of individual portfolio choices among members, we model differences in expected asset returns as Fréchet-distributed shocks around the respective rationally anticipated levels of returns.[8] Borrowing this aggregation technique from the international trade literature (e.g., Eaton and Kortum (2002)), we facilitate the easy incorporation of new asset varieties and distinct degrees of market segmentation across different assets and maturities, while providing an-

[6]Decreases in the aggregate price index further intensify downward pressures on the short-term policy rate under an inflation-targeting policy rule, extending the duration of ZLB episodes.

[7]A parallel finding, albeit through an entirely distinct channel, is presented by Karadi and Nakov (2021). The paper documents the *QE-addiction* problem based on a model incorporating financial frictions, wherein private banks get accustomed to the central bank's liquidity provisions, thereby diminishing their incentive to get recapitalized without additional QE rounds. In that context, Karadi and Nakov (2021) propose a gradual optimal exit strategy from QE programs.

[8]For general properties of the Fréchet distribution, see e.g., Gumbel (1958).

alytically tractable expressions for the household's portfolio shares as functions of relative expected asset returns. Our formulation is highly versatile, encompassing the classic *expectations hypothesis* as a specific case and permitting deviations due to imperfect information and behavioral factors. A final advantage of this framework is that the demand elasticity of each asset class serves as a sufficient statistic for its particular degree of market segmentation, making the segmented market hypothesis easy to test and estimate within the context of our model. We estimate the bond market's segmentation degree or its demand elasticity based on our model structure.

Related Literature The paper contributes to several branches of the macroeconomics and finance literature. Firstly, prior works have demonstrated the significance of macroeconomic factors in elucidating the behavior of the term structure of interest rates (e.g., Ang and Piazzesi (2003), Rudebusch and Wu (2008), and Bekaert et al. (2010)).[9] The models developed within this domain typically employ an ad-hoc affine term structure (e.g., Duffie and Kan (1996)) without micro-foundations.[10] We contribute to the existing literature by examining the term structure of interest rates in the presence of multiple asset classes (e.g., bonds for intertemporal smoothing and private loans for productive investments) and nominal rigidities. Furthermore, explicit consideration of the government's and central bank's balance sheets, along with the household's endogenous portfolio choices across the entire yield curve, enables a comprehensive assessment of the interconnections between business cycle variables, financial markets, and monetary policy.

There are important prior works on the preferred-habitat based theory of the term structure of interest rates, including Modigliani and Sutch (1966), Vayanos and Vila (2021), and Kekre et al. (2023).[11] Ray (2019), based on Vayanos and Vila (2021), proposes a New-Keynesian model that uncovers interesting relationships between monetary policy, business cycles, and the term structure.[12] Our quantitative model generate similar market segmentation based on a new approach, and integrate our term structure of interest rates with real

[9]By examining the joint dynamics of bond yields and macroeconomic variables in a VAR setting, with no-arbitrage as an identifying restriction, Ang and Piazzesi (2003) find models incorporating business cycle factors yield superior forecasts compared to those relying solely on unobservable factors.

[10]Bekaert et al. (2010) integrate the no-arbitrage term-structure within a canonical New-Keynesian model, maintaining the consistency between the household's IS equation and the affine pricing kernel.

[11]In international macroeconomics settings, Gourinchas et al. (2022) and Greenwood et al. (2022) explore the implications of the preferred-habitat setting in jointly determining exchange rates and the term structure of interest rates.

[12]For empirical assessments of the market-segmentation hypothesis as a determinant of the term structure, see e.g., D'Amico and King (2013) and Droste et al. (2021).

economy as well.

Another branch of the literature (e.g., Gertler and Karadi (2011), Cúrdia and Woodford (2011), Christensen and Krogstrup (2018, 2019), Karadi and Nakov (2021)) investigates the relationship between the endogenous size and composition of the central bank's balance sheet and the implications for monetary policy. This literature offers valuable insights into how various large-scale asset purchase programs (LSAPs) employed by central banks can help mitigate various financial market disruptions.[13] However, many of these works do not include multiple bond maturities, focusing instead on the aggregate expansion of the central bank balance sheet. We contribute by presenting a unified framework that describes how central banks can manipulate their bond portfolios to control targeted rates along the yield curve for stabilization purposes. Notably, our finding that active manipulation of the central bank's long-term bond holdings can improve welfare aligns with Sims and Wu (2021).[14]

While our examination of the effects of the zero lower bound aligns closely with prior literature (e.g., Swanson and Williams (2014), Caballero and Farhi (2017), and Caballero et al. (2021)), we highlight the additional advantages of actively managing the central bank's balance sheet (i.e., size and composition along the entire yield curve) when the economy encounters the ZLB. To the best of our knowledge, we are among the first to depict a general equilibrium that incorporates both the term structure of interest rates and the potential for a binding ZLB, alongside the presence of multiple financial assets and the portfolio balance channel.

Layout Section 2 introduces the model and derives the primary theoretical results illustrating how imperfect information results in market segmentation. Section 3 examines the steady-state implications of various policies and model calibration choices. Section 4 explores the cyclical (short-run) responses of our model to distinct shocks under alternative monetary policy regimes and economic contexts, including the ZLB. Section 5 provides concluding remarks. Appendix contains additional figures and tables. Online Appendix A contains detailed derivations and proofs. Online Appendix B explains our calibration and estimation strategies. Online Appendix C derives the second-order approximation to the welfare. In addition, we provide additional figures and corresponding explanations in our

[13] Gertler and Karadi (2011) underscore that (i) central banks are not constrained by their balance sheets, and (ii) as balance sheet constraints on private intermediaries tighten during financial crises, the net benefit from the central bank's intermediation increases. Cúrdia and Woodford (2011) show targeted asset purchases by central banks are effective when financial markets are highly disrupted due to some *exogenous* reason.

[14] Sims and Wu (2021) assume that a wholesale firm and fiscal authorities issue perpetuities with decaying coupon payments.

Supplementary Material.

2 Model

2.1 Representative Household

The representative household maximizes the following objective function:

$$\max_{\{C_{t+j},N_{t+j}\}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\log\left(C_{t+j}\right) - \left(\frac{\eta}{\eta+1}\right) \left(\frac{N_{t+j}}{\bar{N}_{t+j}}\right)^{1+\frac{1}{\eta}} \right] , \tag{1}$$

where $N_t = (\int_0^1 N_t(\nu)^{\frac{\eta+1}{\eta}} \, \mathrm{d}\nu)^{\frac{\eta}{\eta+1}}$ is the aggregate labor index, $N(\nu)$ is the labor supplied to intermediate industry ν, η is the Frisch labor supply elasticity, and $\bar{N}_t$ is the balanced growth path population, which grows at constant gross rate GN. C_t refers to consumption of the final good.

In each period t, the representative household can invest in f-period zero-coupon government bonds with f ranging from 1 to F, and also provide loans to firms.[15,16] As a result, the representative household's period t budget constraint is expressed as follows:

$$C_t + \frac{L_t}{P_t} + \frac{\sum_{f=1}^{F} B_t^{H,f}}{P_t} = \frac{\sum_{f=0}^{F-1} R_t^f B_{t-1}^{H,f+1}}{P_t} + \frac{R_t^K L_{t-1}}{P_t} + \int_0^1 \frac{W_t(\nu)N_t(\nu)}{P_t} \, \mathrm{d}\nu + \frac{\Lambda_t}{P_t} , \tag{2}$$

where L_t is the amount of one-period loans to firms, with associated return R_t^K determined upon issuance. $W_t(\nu)$ is the wage paid by industry ν, and Λ_t includes transfers from various sources, such as the government's lump-sum taxation and profits of the central bank and firms. $B_t^{H,f} \equiv Q_t^f \widetilde{B}_t^{H,f}$ denotes the nominal amount of dollars invested in the f-maturity government bond paying one dollar at the terminal period $t+f$. Q_t^f is the price of such a bond, with Q_t^0 equal to one. $\widetilde{B}_t^{H,f}$ is the amount of f-maturity bonds held by households, and we assume that the households cannot credibly issue risk-free bonds, preventing them from holding a non-negative quantity, $\widetilde{B}_t^{H,f} \geq 0$ for all f. R_t^f is the return earned on an f-period bond, which corresponds to the rate of bond price revaluation between two adjacent

[15]Alternatively, we interpret this as households purchasing one-period corporate bonds

[16]Banks and financial intermediaries are abstracted away in our framework, and the representative household provides direct loans to firms instead. Without any relevant intermediation frictions, the results of both representations are equivalent.

quarters, i.e., $R_t^f = Q_t^f / Q_{t-1}^{f+1}$.

The gross yield of any zero-coupon bond with maturity f is conventionally defined as $YD_t^f \equiv (Q_t^f)^{-\frac{1}{f}}$, allowing us to alternatively express bond return R_t^f as

$$R_t^f = \frac{\left(YD_t^f\right)^{-f}}{\left(YD_{t-1}^{f+1}\right)^{-(f+1)}} .$$

2.1.1 Individual Savings

The representative household determines the optimal levels of consumption, employment, and savings S_t, with the latter allocated into government bonds $B_t^H = \sum_{f=1}^F B_t^{H,f}$ and firm loans L_t, satisfying $S_t = B_t^H + L_t$. To generate a downward-sloping demand curve for each investment vehicle,[17] we introduce the following mechanism: after determining the savings level S_t, the household is equally divided into a $[0,1]$ continuum of families that differ in their preferred savings vehicle, which can be either loans or government bonds. If a family opts to invest in the bond market rather than providing loans, the family is further subdivided into a $[0,1]$ measure of family members, each with a distinct preferred bond maturity $f = 1 \sim F$. We employ index m to identify a family within the continuum, and index n to refer to one of its family members. Each family m and each member n in the bond family m share the same amount of savings S_t as the household. Additionally, within the family, perfect consumption insurance exists and no trading is permitted among members of the same family or different families. We address the allocation problem recursively in the subsequent manner.

Bond family Assuming that a family m selects bonds as its preferred savings vehicle, its member n seeks to maximize the expected savings return by solving the following problem:

$$\max \sum_{f=1}^{F} \mathbb{E}_{m,n,t} \left[Q_{t,t+1} R_{t+1}^{f-1} B_{m,n,t}^{H,f} \right] \text{ s.t. } B_{m,n,t}^{H} \equiv \sum_{f=1}^{F} B_{m,n,t}^{H,f} = S_t, \;\; B_{m,n,t}^{H,f} \geq 0,$$

where $\mathbb{E}_{m,n,t}$ is the expectations operator for member n in family m and $Q_{t,t+1}$ denotes the stochastic discount factor of households. Owing to the problem's linear nature, we arrive at

[17] Otherwise, linearization of the model results in the perfect equalization, in equilibrium, of all expected asset returns (including different bond maturities), which is consistent with the standard expectation hypothesis (see e.g., Froot (1989)).

a corner solution wherein member n allocates her entire share of savings to the bond with the highest expected discounted return.[18] Formally,

$$B^{H,i}_{m,n,t} = \begin{cases} S_t & \text{, if } i = \underset{1\le j\le F}{\arg\max} \left\{\mathbb{E}_{m,n,t}\left[Q_{t,t+1}R^{j-1}_{t+1}\right]\right\} \ , \\ 0 & \text{, otherwise.} \end{cases}$$

In the benchmark rational expectations model, all members in the bond family m select the same allocation, and the expected discounted returns $\mathbb{E}_t\left[Q_{t,t+1}R^{f-1}_{t+1}\right]$ for any maturity f are equalized in equilibrium. This scenario is consistent with the *expectation hypothesis* in the log-linearized economy, where long-term rates are approximated as the average of future expected short-term rates.[19] Since the short-term rate R^0_{t+1} is governed by the central bank, longer yield maturities are entirely determined by conventional monetary policy in this setting. This precludes any significant role for alternative central bank policies, such as quantitative easing (QE), despite empirical evidence to the contrary.[20]

We depart from the expectations hypothesis and generate a downward-sloping demand curve for each bond of arbitrary maturity f by imposing additional structure on the household's portfolio allocation problem. We assume each member n of the family m has different expectations regarding the discounted future returns of bonds. This discrepancy can be attributed to each member possessing access to a distinct and imperfect information set (in a manner akin to Angeletos and La'O (2013)) or simply to behavioral assumptions. Furthermore, we assume that family m lacks the capacity to aggregate individual information from its members and execute a centralized portfolio allocation based on signal extraction. Consequently, the family opts to evenly divide the savings among its members and permits them to decide on the allocation of their individual share. We assume the following functional form for member n expectations:

$$\mathbb{E}_{m,n,t}\left[Q_{t,t+1}R^{f-1}_{t+1}\right] = z^f_{n,t}\cdot\mathbb{E}_t\left[Q_{t,t+1}R^{f-1}_{t+1}\right],\ \ \forall f = 1,\ldots,F,$$

where operator $\mathbb{E}_{m,n,t}$ represents a member-specific expectation, whereas $\mathbb{E}_t$ denotes the

[18] An exception would occur if two or more bonds possess precisely the same highest expected discounted return, in which case member n would be indifferent between allocations across these bonds. This scenario, in general, will happen with zero probability, as will become evident in the derivations below.

[19] In the linearized economy, the covariance between $Q_{t,t+1}$ and returns R^{f-1}_{t+1} is omitted along with other higher-order effects. Consequently, expected returns for each bond maturity are equalized.

[20] For instance, Krishnamurthy and Vissing-Jorgensen (2011) demonstrate that large-scale asset purchase (LSAP) interventions reduce long-term interest rates.

rational expectations operator. $z_{t,n}^{f}$ is maturity-f specific shock to member n's expectations. Observe that, *ceteris paribus*, a high realization of $z_{t,n}^{f}$ renders member n more inclined to save in the f-maturity bond.

For analytical tractability, we model $z_{t,n}^{f}$ as a Fréchet-distributed shock with location parameter zero, scale parameter z_t^f, and shape parameter κ_B, assuming it to be independent and identically distributed across members n, maturities f, and quarters t.[21] The shape parameter κ_B governs the volatility of these expectation shocks, with $\lim_{\kappa_B \to \infty} Var\left(z_{t,n}^{f}\right) = 0$. Consequently, when $z_t^f = \Gamma\left(1 - \frac{1}{\kappa_B}\right)^{-1}$ [22] and $\kappa_B \to \infty$, the model converges to the standard rational expectations case, with $\mathbb{E}_{m,n,t}$ coinciding with $\mathbb{E}_t$. In other cases, individual expectations deviate from the rational expectation.[23]

We define $\lambda_t^{HB,f}$ as the probability that the f-period bond offers the highest expected discounted return for an individual n within the family. Owing to the characteristics of the Fréchet distribution, we derive the following expression for this probability:

$$\lambda_t^{HB,f} = \left(\frac{z_t^f \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{f-1} \right]}{\Phi_t^B} \right)^{\kappa_B}, \tag{3}$$

where $\Phi_t^B \equiv \left[\sum_{j=1}^{F} \left(z_t^j \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{j-1} \right] \right)^{\kappa_B} \right]^{\frac{1}{\kappa_B}}$ is an aggregate index capturing the average expected discounted return across bonds of varying maturities. In (3), it can be observed that the demand for savings in an f-maturity bond increases when f-maturity return, R_{t+1}^{f-1}, is comparatively higher than the mean bond return across all maturities, Φ_t^B. Furthermore, the scale parameter z_t^f dictates the overall portfolio preference of households for the specific maturity f. For instance, a sudden increase in z_t^1 amplifies the household's demand for the shortest term (i.e., $f = 1$) bond, irrespective of relative holding returns.

By aggregating across families and individual family members, we arrive at an expression representing the total holdings of the household for each f-maturity bond:

$$B_t^{H,f} = \lambda_t^{HB,f} \cdot B_t^H, \;\; \forall f = 1, \dots, F, \tag{4}$$

[21] See, for example, Eaton and Kortum (2002) and Dordal i Carreras et al. (2023) for applications of the Fréchet distribution and aggregation issues in the international trade and macroeconomics literature.

[22] Setting the scale parameter to $z_t^f = \Gamma\left(1 - 1/\kappa_B\right)^{-1}$, we obtain $\mathbb{E}(z_{n,t}^{f}) = 1$, and the member-specific expectations fluctuate around the rational expectation.

[23] Hence, our framework encompasses the aforementioned benchmark case (no-arbitrage term structure) as a limiting case.

where B_t^H denotes the aggregate bond holding amounts for the household. Using equation (4), we derive an aggregate expression for the returns on the household's bond portfolio as

$$R_{t+1}^{HB} = \sum_{f=0}^{F-1} \lambda_t^{HB,f+1} R_{t+1}^{f}. \tag{5}$$

Bond vs. loans family Having determined the allocation of savings across bond maturities, we now examine how a family m chooses between allocating its savings to bonds or loans. Family m seeks to maximize savings returns from the set of possible asset classes (i.e., bonds and loans in our model) by solving the following optimization problem:

$$\max\ \mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^{HB}B_{m,t}^{H}\right] + \mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^{K}L_{m,t}\right] \quad \text{s.t.}$$
$$B_{m,t}^{H} + L_{m,t} = S_t,\ \ B_{m,t}^{H} \geq 0,\ \ \text{and}\ \ L_{m,t} \geq 0.$$

Family m assumes that if it becomes a bond family, it will adhere to the investment strategy delineated in (3) and earn the aggregate bond returns R_{t+1}^{HB} (i.e., (5)) on its bond portfolio. In the benchmark rational expectations environment, all families select the same allocation, and at equilibrium, expected discounted returns $\mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^{HB}\right]$ and $\mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^{K}\right]$ are equalized, rendering families indifferent in their portfolio allocation. We depart from this scenario by assuming that each family m's expectation deviates from the rational expectation as follows:

$$\mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^{K}\right] = z_{m,t}^{K} \cdot \mathbb{E}_{t}\left[Q_{t,t+1}R_{t+1}^{K}\right],$$

where $\mathbb{E}_t$ represents the rational expectations and $\mathbb{E}_{m,t}$ is a family m-specific expectation. We model $z_{m,t}^{K}$ as a Fréchet-distributed shock with location parameter of zero, scale parameter z_t^K, and shape parameter κ_S, assuming $z_{m,t}^{K}$ to be independent and identically distributed across families m and quarters t. As before, κ_S controls the expectation shock's volatility, with $\lim_{\kappa_S\to\infty} \mathrm{Var}\left(z_{m,t}^{K}\right) = 0$. Consequently, when $z_t^K = \Gamma\left(1 - 1/\kappa_S\right)^{-1}$ and $\kappa_S \to \infty$ the model converges to the standard rational expectations case, wherein $\mathbb{E}_{m,t}$ aligns with $\mathbb{E}_t$. Building on our previous findings, we can now aggregate the decisions of each family m to determine the share of aggregate savings allocated to loans as

$$\lambda_t^K = \left(\frac{z_t^K \mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{K}\right]}{\Phi_t^S}\right)^{\kappa_S}, \tag{6}$$

where $\Phi_t^S = \left[\left(\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{HB}\right]\right)^{\kappa_S} + \left(z_t^K\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{K}\right]\right)^{\kappa_S}\right]^{\frac{1}{\kappa_S}}$ is the aggregate index capturing the average expected discounted return of bonds and loans.[24] The scale parameter z_t^K governs the overall portfolio preference of households for private loans. For instance, a sudden increase in z_t^K raises the share of loans irrespective of the relative savings returns.

Using (6), we can now express the aggregate amount of savings allocated to bonds of each maturity as:

$$B_t^{H,f} = \left(1-\lambda_t^K\right)\cdot\lambda_t^{HB,f}\cdot S_t,\ \ \forall f=1,\ldots,F,$$

and the aggregate return on household savings as:

$$R_t^S = \left(1-\lambda_{t-1}^K\right)R_t^{HB} + \lambda_{t-1}^K R_t^K. \tag{7}$$

Note that R_t^S depends on the rates of all available assets, encompassing (i) different bond maturities and (ii) private loans, with endogenous weights determined by the relative returns of these assets. Lastly, we can rewrite the budget constraint in (2) as:

$$C_t + \frac{S_t}{P_t} = \frac{R_t^S S_{t-1}}{P_t} + \int_0^1 \frac{W_t(\nu)N_t(\nu)}{P_t}\,\mathrm{d}\nu + \frac{\Lambda_t}{P_t}\,. \tag{8}$$

It is worth noting that the representative household problem now resembles that of a conventional New-Keynesian model, despite the asset variety and market segmentation introduced.

Remarks on aggregation: The assumption regarding separate information sets on asset returns, which we model as extreme type Fréchet deviations from the rational expectations equilibrium, effectively creates market segmentation (i) between bond and loan markets, and (ii) among bonds of different maturities. The literature also empirically supports this result (see, for example, D'Amico and King (2013)). The shape parameters (κ_B, κ_S) control the degree of market segmentation across maturities and assets, respectively, and the conventional expectations hypothesis framework without market segmentation is nested as a special case of our model when $\kappa_B,\ \kappa_S \to \infty$. Most notably, the nested CES structure of our asset markets can be easily extended to accommodate a wide variety of assets and maturity structures. Shape parameters (κ_B, κ_S) summarize the demand elasticity for financial

[24] Note that (6) implies that a family's preference for issuing loans increases when the return on loans R_{t+1}^K becomes relatively higher than that of the aggregate bond portfolio R_{t+1}^{HB}.

products in response to movements in their expected returns, as seen in (3) and (6). These elasticities can take distinct values across asset classes and can be easily estimated from data to capture different degrees of market segmentation across assets and maturities. We estimate κ_B in Appendix B, based on (3): the household's bond portfolio depends on the current maturity preference (i.e., z^f) shocks as well as relative holding returns of different maturities bonds, with κ_B as the elasticity.

2.1.2 Optimality Conditions

The solution to the household's problem in (1) subject to the budget constraint in (8) yields the following equilibrium conditions:

$$\left(\frac{N_t(\nu)}{\bar{N}_t}\right)^{\frac{1}{\eta}} = \left(\frac{C_t}{\bar{N}_t}\right)^{-1}\frac{W_t(\nu)}{P_t}, \tag{9}$$

$$1 = \beta\mathbb{E}_t\left[\frac{R^S_{t+1}C_t}{C_{t+1}\Pi_{t+1}}\right], \tag{10}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the gross inflation rate. Note that in the Euler equation (10), the effective savings rate R^S_{t+1} is the reference rate for the household's intertemporal consumption decisions. Note that the household as a whole follows the rational expectations.

2.2 Capital Producer

There exists a representative firm that produces capital K_t and rents it to intermediate good producers at price P^K_t. The capital is produced by utilizing the final good as an investment input, depreciates at rate δ, and there is a one-period lag for investment I_t to be deployed as new capital. Consequently, the evolution of capital is defined as

$$K_t = (1-\delta)K_{t-1} + I_{t-1}.$$

The profits of the capital producer are given by

$$\Lambda^K_t = P^K_t K_t - P_t I_t,$$

where P_t denotes the price index of the final good. Solving the capital producer's optimization problem with respect to I_t, we derive the following first-order condition:

$$1 = \mathbb{E}_t \left[Q_{t,t+1} \Pi_{t+1} \left[(1-\delta) + \frac{P^K_{t+1}}{P_{t+1}} \right] \right] . \tag{11}$$

2.3 Firms

There exists a continuum $\nu \in [0,1]$ of intermediate goods, where each ν is produced by a monopolist ν utilizing capital and labor according to the following production function:

$$Y_t(\nu) = \left(\frac{K_t(\nu)}{\alpha} \right)^{\alpha} \left(\frac{A_t N_t(\nu)}{1-\alpha} \right)^{1-\alpha} , \tag{12}$$

where $A_t = \exp\left(u^A_t\right)$ represents the aggregate technology, with $u^A_t = \mu + u^A_{t-1} + \varepsilon^A_t$, $\varepsilon^A_t \sim N\left(0, \sigma^2_A\right)$. We define GA_t as the (gross) rate of change in A_t, thus $GA_t \equiv \frac{A_t}{A_{t-1}} = \exp(\mu + \varepsilon^A_t)$ holds true.

A representative, perfectly competitive firm aggregates all intermediate products into a final good according to the well-known Dixit-Stiglitz aggregator, as follows:

$$Y_t = \left[\int_0^1 Y_t(\nu)^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}\nu \right]^{\frac{\epsilon}{\epsilon-1}} ,$$

where $\epsilon > 1$ is the elasticity of substitution between varieties. The household's demand for intermediate good ν is given by

$$Y_t(\nu) = \left(\frac{P_t(\nu)}{P_t} \right)^{-\epsilon} Y_t , \tag{13}$$

where $P(\nu)$ represents the price of intermediate ν. The aggregate price index is given by

$$P_t = \left[\int_0^1 P_t(\nu)^{1-\epsilon} \, \mathrm{d}\nu \right]^{\frac{1}{1-\epsilon}} . \tag{14}$$

Intermediate producers exhibit sticky prices à la Calvo (1983), resetting their prices at the beginning of each quarter with probability $1-\theta$. All price-changing firms reset their prices to the same optimal price (in equilibrium) within a given period, denoted by P^*_t. This enables us to $P_t^{1-\epsilon} = (1-\theta)\left(P^*_t\right)^{1-\epsilon} + \theta \left(P_{t-1}\right)^{1-\epsilon}$.

Intermediate producers rent capital at price P^K_t, subsequently paying $P^K_t K_t(\nu)$ to the capital producer at quarter t. As firm profits are rebated to the representative household at the end of every quarter, firms are financially constrained. For simplicity, we assume that each firm ν borrows γ portion of the revenue it would generate, i.e., $(1+\zeta^F)P_t(\nu)Y_t(\nu)$ where ζ^F is a production subsidy, from households. Formally, if firm ν borrows $L_t(\nu) = \gamma(1+\zeta^F)P_t(\nu)Y_t(\nu)$ from the household, it repays $R^K_{t+1}L_t(\nu)$ to the household in period $t+1$, with the rate R^K_{t+1} contracted at period t.

An intermediate firm ν seeks to maximize the discounted stream of profits, solving

$$\max \sum_{j=0}^{\infty} \mathbb{E}_t[Q_{t,t+j}[(1+\zeta^F)P_{t+j}(\nu)Y_{t+j}(\nu) - W_{t+j}(\nu)N_{t+j}(\nu) - P^K_{t+j}K_{t+j}(\nu) \\ - R^K_{t+j}L_{t+j-1}(\nu) + L_{t+j}(\nu)]] \tag{15}$$

where $Q_{t,t+j} = \beta^j \left(\frac{P_{t+j}}{P_t} \cdot \frac{C_{t+j}}{C_t}\right)^{-1}$ is households' stochastic discount factor (SDF) between periods t and $t+j$, and $L_{t+j} = \gamma(1+\zeta^F)P_{t+j}(\nu)Y_{t+j}(\nu)$ for $\forall j$. It is important to note that at period $t+j$, firm ν repays $R^K_{t+j}L_{t+j-1}(\nu)$ to the household, as it received a loan amounting to $L_{t+j-1}(\nu)$ in the previous period $t+j-1$.

Minimizing a firm ν's production costs with respect to labor and capital, we derive the following demand for inputs:

$$N_t(\nu) = (1-\alpha)\frac{Y_t(\nu)}{A_t}\left(\frac{\frac{P^K_t}{P_t}}{\frac{W_t(\nu)}{P_tA_t}}\right)^{\alpha}, \quad \frac{K_t(\nu)}{A_t} = \alpha\frac{Y_t(\nu)}{A_t}\left(\frac{\frac{P^K_t}{P_t}}{\frac{W_t(\nu)}{P_tA_t}}\right)^{-(1-\alpha)}. \tag{16}$$

The aggregate profit of firms is rebated to the household and expressed as

$$\Lambda^F_t = (1+\zeta^F)P_tY_t - \int_0^1 W_t(\nu)N_t(\nu)\,\mathrm{d}\nu - P^K_{t-1}K_{t-1} - R^K_tL_{t-1} + L_t$$

2.4 Bond Market

The equilibrium condition in the bond market can be expressed as

$$B^{H,f}_t + B^{G,f}_t + B^{CB,f}_t = 0, \ \ \forall f = 1, \dots, F, \tag{17}$$

where $B^{G,f}_t$ and $B^{CB,f}_t$ represent 'nominal' bonds held by the government[25] and the central bank, respectively. We assume that both the government and the central bank are the sole

[25]Given our assumption that the government issues bonds across different maturities, $B^{G,f}_t \leq 0$ for all $f = 1 \sim F$.

agents in the economy capable of issuing riskless claims and thus holding negative bond positions. For the central bank, a negative bond position can be interpreted as permitting interest-bearing excess reserves, as observed following the Great Recession.[26] We allow the government to issue bonds, and thereby hold negative positions in bonds of each maturity, as risk-free assets would not exist otherwise. Specifically, our specification of technology growth GA_t and population growth GN ensures that, at the steady state, the government maintains a non-zero (and non-explosive) amount of debt obligations and consistently acts as the supplier of risk-free debt despite cyclical fluctuations.

By defining $\lambda_t^{G,f}$ and $\lambda_t^{CB,f}$ as the shares of nominal f-maturity bond holdings of the government and the central bank, respectively, (17) can be written as

$$\lambda_t^{HB,f} B_t^H + \lambda_t^{G,f} B_t^G + \lambda_t^{CB,f} B_t^{CB} = 0, \;\; \forall f = 1, \dots, F. \tag{18}$$

2.5 Government

The government's budget constraint is represented by

$$G_t + \zeta^F Y_t + \frac{B_t^G}{P_t} = T_t + \frac{R_t^G B_{t-1}^G}{P_t}, \;\; \text{with } B_t^G = \sum_{f=1}^{F} B_t^{G,f}, \;\; R_t^G = \sum_{f=0}^{F-1} \lambda_{t-1}^{G,f+1} R_t^f, \tag{19}$$

where B_t^G denotes the government's nominal bond position, G_t signifies the real government spending, T_t represents taxes, and R_t^G refers to the aggregate bond return to the government's portfolio $\{B_{t-1}^{G,f}\}_{f=1}^F$, with $\lambda_t^{G,f}$ being the fraction of government debt obligations outstanding as an f-maturity bond. Formally, we have $B_t^{G,f} = \lambda_t^{G,f} \cdot B_t^G$, $\forall f = 1, \dots, F$, where both $\lambda_t^{G,f}$ and B_t^G are exogenous.[27] The budget constraint in (19) can be rewritten as

$$\frac{B_t^G}{P_t} = \frac{R_t^G B_{t-1}^G}{P_t} - \left[\zeta_t^G + \zeta^F - \zeta_t^T\right] Y_t \,, \tag{20}$$

where $\zeta_t^G = \frac{G_t}{Y_t}$ and $\zeta_t^T = \frac{T_t}{Y_t}$ are the government spending and taxation as shares of GDP, respectively. Both variables, ζ_t^G and ζ_t^T, are also exogenous within our framework.

[26] For theoretical and empirical analyses of the roles of excess reserves in conjunction with the federal fund market and interbank credit markets in general, see, for example, Frost (1971), Güntner (2015), Mattingly and Abou-Zaid (2015), Primus (2017), and Ennis (2018).

[27] We abstract from the government's optimal maturity structure problem and assume its gross bond positions and portfolios across maturities are exogenous, focusing on the central bank's monetary policy.

2.6 Central Bank

The profits generated by the bonds held on the central bank's balance sheet can be expressed as

$$\Lambda_t^{CB} = R_t^{CB} B_{t-1}^{CB} - B_t^{CB}, \text{ with } B_t^{CB} = \sum_{f=1}^{F} B_t^{CB,f}, \; R_t^{CB} = \sum_{f=0}^{F-1} \lambda_{t-1}^{CB,f+1} R_t^f, \tag{21}$$

where B_t^{CB} represents the central bank's total nominal bond position across different maturities, and R_t^{CB} denotes the aggregate index of bond returns to the central bank's portfolio $\{B_{t-1}^{G,f}\}_{f=1}^{F}$. The fraction of the central bank's bonds held at maturity f is given by $\lambda_t^{CB,f}$. Formally, we have

$$B_t^{CB,f} = \lambda_t^{CB,f} \cdot B_t^{CB}, \; \forall f = 1, \ldots, F, \tag{22}$$

where B_t^{CB} and $\lambda_t^{CB,f}$ are dependent on the monetary policy rules, which will be described shortly. The central bank's profit at time t, i.e., Λ_t^{CB} in (21), is transferred as a lump sum money to the household, constituting a part of the total transfer Λ_t in (2).

2.7 Monetary Policy

Since the above equation (22) introduces F new equations to the model, the central bank's monetary policy has F degrees of freedom, which must be filled in for the model to achieve a determinate nominal equilibrium.[28] Monetary authorities may opt for one of the following policy implementations:

1. For any f-maturity bond, establish a rule on $B_t^{CB,f}$. Consequently, the f-maturity bond's prices (and yields) adjust.
2. For any f-maturity bond, establish a rule on its yield YD_t^f (or equivalently, its price Q_t^f). Then, adjust the purchase amounts of the f-maturity bond $B_t^{CB,f}$ to accordingly alter the yield.
3. A combination of the previous two policies at different maturities.

The first Case 1 resembles textbook money supply rules, where central banks control long-term bond supplies rather than money. Case 2 exemplifies a policy approach often referred

[28] The central bank chooses its bond portfolio across different maturities $\{\lambda_t^{CB,f}\}_{f=1}^{F}$, as well as its gross debt position B_t^{CB}. The central bank's portfolio problem is typically abstracted away in the New-Keynesian models as the explicit term structure of interest rates is usually absent.

to as yield-curve control (YCC), which Japan implemented in 2016.[29] Case 3 is a mixture of the previous options and includes widely employed rules such as the traditional short-term rate target of conventional monetary policy, as will be discussed below.

In this paper, we aim to study the distinct economic and welfare implications of conventional and unconventional policy interventions. The specific implementation of the latter type of policies can potentially adopt any of the three cases considered. For simplicity, we assume that the fundamental trait characterizing unconventional interventions (e.g., QE and LSAP) is their intent to affect asset returns along the entire yield curve (as opposed to conventional policy focused on short-term rates). Therefore, we adopt a YCC policy rule as the representative unconventional policy within our framework. In the following, we formally characterize the equations describing conventional and YCC policy rules.

Conventional policy The conventional monetary policies targeting the short-term interest rate align with Case 3, in which the central bank establishes a rule on YD_t^1 without manipulating longer-term bonds. We assume that the central bank maintains its (normalized)[30] positions for long-term bonds as follows:

$$R_{t+1}^0 \equiv YD_t^1 = \max\left\{YD_t^{1*}, 1\right\}, \tag{23a}$$

$$\frac{YD_t^{1*}}{\overline{YD}^1} = \left(\frac{YD_{t-1}^{1*}}{\overline{YD}^1}\right)^{\rho_1} \left(\frac{YD_{t-2}^{1*}}{\overline{YD}^1}\right)^{\rho_2} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi} \left(\frac{Y_t}{\bar{Y}_t}\right)^{\gamma_y} \exp\left(\tilde{\varepsilon}_t^{YD^1}\right)\right]^{1-(\rho_1+\rho_2)}, \tag{23b}$$

$$\frac{B_t^{CB,f}}{A_t \bar{N}_t P_t} = \frac{\overline{B^{CB,f}}}{A\bar{N}P} \ \forall f = 2, \ldots, F, \tag{23c}$$

where YD_t^{1*} follows a standard Taylor rule targeting inflation and output deviations, with $\tilde{\epsilon}_t^{YD^1}$ representing a monetary policy shock. When YD_t^{1*} falls below 1, the monetary policy is constrained by the ZLB, resulting in $R_{t+1}^0 \equiv YD_t^1 = 1$, as implied by equation (23a).[31]

[29] On September 21, 2016, the Bank of Japan (BOJ) combined a new long-term rate target with its existing short-term rate target to implement its own version of yield-curve-control policies. The Bank of Japan (BOJ) set its short-term policy target, the rate paid on bank reserves, at -0.1% and capped its long-term target rate, that on 10-year government bonds, at approximately zero. For the case of the United States, see Humpage (2016) for the Fed's yield-curve-control policy in the WW2 era and the policy's benefits and costs.

[30] We define a normalized variable as the variable adjusted for technology, population, and price growth (in the case of nominal variables).

[31] For instance, a sudden increase in the preference parameter z_t^1 might prompt households to raise the demand for the shortest-term bond, driving its yield towards zero. The shock potentially leads to recessionary pressures as households reduce consumption when the ZLB is reached.

Yield-Curve-Control policy In a yield-curve-control policy, the central bank targets the entire yield curve by implementing a Taylor rule for each bond maturity, as follows:

$$YD_t^{YCC,1} = \max\left\{YD_t^{1*}, 1\right\}, \tag{24a}$$

$$\frac{YD_t^{1*}}{\overline{YD}^1} = \left(\frac{YD_{t-1}^{1*}}{\overline{YD}^1}\right)^{\rho_1}\left(\frac{YD_{t-2}^{1*}}{\overline{YD}^1}\right)^{\rho_2}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi^1}\left(\frac{Y_t}{\bar{Y}_t}\right)^{\gamma_y^1}\exp\left(\tilde{\varepsilon}_t^{YD^1}\right)\right]^{1-(\rho_1+\rho_2)}, \tag{24b}$$

$$\frac{YD_t^{f*}}{\overline{YD}^f} = \left(\frac{YD_{t-1}^{f*}}{\overline{YD}^f}\right)^{\rho_1}\left(\frac{YD_{t-2}^{f*}}{\overline{YD}^f}\right)^{\rho_2}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi^f}\left(\frac{Y_t}{\bar{Y}_t}\right)^{\gamma_y^f}\exp\left(\tilde{\varepsilon}_t^{YD^f}\right)\right]^{1-(\rho_1+\rho_2)}, \tag{24c}$$

$$YD_t^{YCC,f} = \overline{YD}^f\left(\frac{YD_t^{CP,f}}{\overline{YD}^f}\right)^{\gamma_{CP}^f}\left[\frac{YD_t^{f*}}{\overline{YD}^f}\right]^{1-\gamma_{CP}^f}, \tag{24d}$$

for $f \geq 2$, where γ_π^f and γ_y^f represent the responsiveness to inflation and output deviations across maturities $f = 1 \sim F$, respectively, with $\tilde{\varepsilon}_t^{YD^f}$ denoting a monetary policy shock to a f-maturity bond yield. The term $YD_t^{CP,f}$ refers to the f-maturity yield that will prevail in a counterfactual economy under the conventional monetary policy in (23). The parameter $\gamma_{CP}^f \in [0,\, 1]$ enables some control over the influence of conventional policy targets under our yield-curve-control regime. When $\gamma_{CP}^f = 1$, equation (24) reverts to the conventional policy regime, while $\gamma_{CP}^f = 0$ corresponds to the pure yield-curve-control case, where the yield for the maturity-f bond is given by:

$$YD_t^{YCC,f} = \overline{YD}^f\left(\frac{YD_{t-1}^{f*}}{\overline{YD}^f}\right)^{\rho_1}\left(\frac{YD_{t-2}^{f*}}{\overline{YD}^f}\right)^{\rho_2}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi^f}\left(\frac{Y_t}{\bar{Y}_t}\right)^{\gamma_y^f}\exp\left(\tilde{\varepsilon}_t^{YD^f}\right)\right]^{1-(\rho_1+\rho_2)}.$$

In this case, the central bank does not take into account concerns related to balance sheet exposure for bonds with maturity f (i.e., equation (23c)). Intermediate values, $0 < \gamma_{CP}^f < 1$, correspond to scenarios in which the monetary authority balances intervention in the yield curve with balance sheet composition and size. For simplicity, in the subsequent exercises, we consider $\gamma_{CP}^f = 0$ for $\forall f$ as the representative form of unconventional monetary policy.

2.8 Market Clearing

Using the bond market equilibrium (i.e., (17)), the total transfers to households from firms, the central bank, capital producers, and the government are:

$$\Lambda_t = P_t Y_t - P_t G_t - P_t I_t - \int_0^1 W_t(\nu) N_t(\nu)\,\mathrm{d}\nu - R_t^K L_{t-1} + L_t + B_t^H - R_t^H B_{t-1}^H \,. \tag{25}$$

where we defined $\Lambda_t \equiv \Lambda_t^F + \Lambda_t^{CB} + \Lambda_t^K - P_t T_t$. By combining (25) with the household's budget constraint (i.e., (2)), the standard aggregate market clearing condition can be derived:

$$C_t + G_t + I_t = Y_t, \tag{26}$$

2.9 Aggregation

Aggregating labor demand (i.e., (16)) across firms yields:

$$\frac{N_t}{\bar{N}_t} = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{C_t}{A_t \bar{N}_t}\right)^{-\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t \bar{N}_t}\right)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{P_t^K}{P_t}\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}}, \tag{27}$$

where Δ_t is a measure of price dispersion, recursively defined as:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta \Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \Delta_{t-1}. \tag{28}$$

Notice from (27), that the labor N_t supporting a given (normalized) level of consumption and output increases with price dispersion Δ_t, which is a proxy for the inefficiency caused by nominal rigidities. An increase in $\frac{P_t^K}{P_t}$ raises the rental cost of capital, inducing firms to substitute capital with labor and raising N_t. This channel is also observable in the following aggregate capital equilibrium condition,[32]

$$\frac{K_t}{A_{t-1}\bar{N}_{t-1}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \cdot GA_t \cdot GN \cdot \left(\frac{C_t}{A_t \bar{N}_t}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_t}{A_t \bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\frac{P_t^K}{P_t}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \Delta_t. \tag{29}$$

Aggregate capital K_t rises when consumption, output, or price dispersion increase and/or the rental price of capital decreases. Thus, the above two equations emphasize the role of firms' substitution between capital and labor during the production stage.[33]

[32] We normalize K_t by $A_{t-1}\bar{N}_{t-1}$ as K_t is determined at $t-1$, while the aggregate labor N_t, consumption C_t, and output Y_t are all normalized by $A_t\bar{N}_t$, and the rental price of capital P_t^K is normalized by the nominal price index P_t.

[33] The elasticity of substitution between capital and labor is 1 due to Cobb-Douglas production function.

As the supply block (i.e., price-resetting decisions of firms, where price-resetting firms solve the problem in (15)) resembles that of a standard New-Keynesian model,[34] our focus shifts to the demand block. The representative household satisfies the Euler equation (i.e., (10)), where R^S_{t+1} satisfies (7), λ^K_t satisfies (6), and $\lambda^{HB,f}_t$ is given by (3).

The equilibrium condition for the household's allocation between loans and bonds can be expressed as:

$$\frac{L_t}{B^H_t} = \frac{(1+\zeta^F)P_tY_t}{B^H_t} = \frac{\lambda^K_t}{1-\lambda^K_t}, \tag{30}$$

where $B^H_t = \sum_{f=1}^F B^{H,f}_t$ and $L_t = (1+\zeta^F)\int P_t(\nu)Y_t(\nu)d\nu = (1+\zeta^F)P_tY_t$ represent the aggregate household bond and loan holdings, respectively.

2.9.1 Conventional Policy

In the case of conventional policy (i.e., (23)), the monetary authority does not manipulate its normalized long-term maturity bond holdings; thus, $\frac{B^{CB,f}_t}{A_t\bar{N}_tP_t}$ remains constant for $f > 1$. Under this scenario, $\lambda^{HB,f}_t$ must satisfy:

$$\lambda^{HB,f}_t = -\frac{\frac{B^{G,f}_t}{A_t\bar{N}_tP_t} + \overline{\frac{B^{CB,f}}{A\bar{N}P}}}{\frac{B^H_t}{A_t\bar{N}_tP_t}}, \quad \forall f > 1. \tag{31}$$

2.9.2 Yield-Curve-Control Policy

In the yield-curve-control case (i.e., (24)), monetary policy affects households' bond portfolio across maturities $\{\lambda^{HB,f}_t\}_{f=1}^F$ and the effective bond rate R^{HB}_t through the following relations:

$$\lambda^{HB,f}_t = \left(\frac{z^f_t\mathbb{E}_t\left[Q_{t,t+1}R^{f-1}_{t+1}\right]}{\Phi^B_t}\right)^{\kappa_B}, \quad R^{f-1}_{t+1} = \frac{\left(YD^{f-1}_{t+1}\right)^{-(f-1)}}{\left(YD^f_t\right)^{-f}}. \tag{32}$$

A change in the household's bond portfolio across maturities $\{\lambda^{HB,f}_t\}_{f=1}^F$ results in changes in the effective bond rate of households R^{HB}_t through (5), the loan rate R^K_t through (6), the effective savings rate R^S_t through (7), consumption through the Euler equation in (10), and other aggregate outcomes through (26), (27), (29), and (30).

[34] The optimal pricing decisions for firms (i.e., (15)) and the aggregation are derived in Appendix A.

2.10 Shock Processes

We assume that both Fréchet distribution scale parameters, $\{z_t^f\}_{f=1}^F$ and z_t^K, follow $AR(1)$ processes, satisfying $z_t^f = \rho_z z_{t-1}^f + \varepsilon_t^{z,f}$, with $\mathrm{Var}(\varepsilon_t^{z,f}) = (\sigma_z)^2$, and $z_t^K = \rho_z^K z_{t-1}^K + \varepsilon_t^{z,K}$, with $\mathrm{Var}(\varepsilon_t^{z,K}) = (\sigma_z^K)^2$.

For the government spending ratio $\zeta_t^G = \frac{G_t}{Y_t}$ and revenue ratio $\zeta_t^T = \frac{T_t}{Y_t}$, we assume the following shock processes:

$$\zeta_t^G = \frac{1}{1 + a^G \exp\left(-u_t^G\right)}, \quad \zeta_t^T = \frac{1}{1 + a^T \exp\left(-u_t^T\right)}. \tag{33}$$

where a^G and a^T are constants, and u_t^G and u_t^T follow standard $AR(1)$ processes, given as $u_t^G = \rho_G u_{t-1}^G + \varepsilon_t^G$, $u_t^T = \rho_T u_{t-1}^T + \varepsilon_t^T$, with ε_t^G and ε_t^T being i.i.d shocks.

Since the government's bond shares across maturities $\{\lambda_t^{G,f}\}_{f=1}^F$ are exogenously given, we specify their processes as follows:

$$\lambda_t^{G,1} = \frac{1}{1 + \sum_{l=2}^{F} a^{B,l} \exp\left(\tilde{u}_t^{B,l}\right)}, \quad \lambda_t^{G,f} = \frac{a^{B,f} \exp\left(\tilde{u}_t^{B,f}\right)}{1 + \sum_{l=2}^{F} a^{B,l} \exp\left(\tilde{u}_t^{B,l}\right)}, \ \forall f > 1, \tag{34}$$

where $a^{B,f}, \ \forall f > 1$ are constants. When F is large, we reduce the number of independent shocks to $J \leq F$ by assuming

$$\tilde{u}_t^{B,f} = \sum_{j=2}^{J} \tau_{fj}^B u_t^{B,j}, \tag{35}$$

where $u_t^{B,j}$ follows an independent $AR(1)$ process and τ_{fj}^B is a constant, for $\forall j, f$. Similarly, we can also reduce the state-space of shocks in the yield-curve-control regime by assuming that monetary policy shocks $\{\tilde{\varepsilon}_t^{YD^f}\}$ to different maturities $\forall f$ can be represented as linear combinations of several factors, as[35]

$$\tilde{\varepsilon}_t^{YD^f} = \sum_{l=1}^{L} \tau_{f,l}^{YD} \varepsilon_t^{YD^l}, \tag{36}$$

where $\tau_{f,l}^{YD}, \ \forall l, f$ are constants, $\varepsilon_t^{YD^l}$ are i.i.d. shocks, and $L \leq F$. Figure 1.2 in Appendix

[35] In estimating ρ_B and $\sigma^{B,j}$ in (35), we employ the principal component analysis (PCA) on the time-series data of government bond portfolio shares, focusing on the 7 most prominent components.

provides a graphical illustration of the model. All other equilibrium conditions are provided in Online Appendix A.

3 Steady-State (Long-Run) Analysis

3.1 Steady-State Relations

At the steady state, the central bank chooses the level of holdings for the f-maturity bond, i.e., $B^{CB,f} = \lambda^{CB,f} B^{CB}$. We assume that the total bond holding of the central bank, B^{CB}, is a ζ^{CB} fraction of the total government bond holding B^G, i.e., $B^{CB} = \zeta^{CB} B^G$.[36] Given $\left\{\lambda^{CB,f}\right\}_{f=1}^{F}$, the steady-state relation in the Treasury market (i.e., (18)) can be expressed as:

$$\lambda^{HB,f} = \frac{\lambda^{G,f} + \lambda^{CB,f}\zeta^{CB}}{1+\zeta^{CB}}.$$

Hence, the steady-state household's bond portfolio shares across maturities, $\{\lambda^{HB,f}\}_{f=1}^{F}$, are determined by exogenous parameters, $\{\lambda^{G,f}, \lambda^{CB,f}\}_{f=1}^{F}$ and ζ^{CB}.

In the steady state, the government's budget constraint, as represented by equation (20), can be expressed as:

$$\frac{B^G}{A\bar{N}P} = -\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1} \left[\zeta^G + \zeta^F - \zeta^T\right] \frac{Y}{A\bar{N}}.$$

Given the normalized output level $\frac{Y}{A\bar{N}}$ and a positive primary deficit ratio $\zeta^G + \zeta^F - \zeta^T > 0$, an increase in the interest rate on government debt, R^G, results in a higher volume of bond issuance, $|B^G|$, (since $B^G < 0$), and a higher debt-to-output ratio. This occurs because the government must pay more in the form of interest from its own debt position[37].

The remaining steady-state relationships and procedures for characterizing these conditions are detailed in Online Appendix A.1.3.

[36]Given that $B^G < 0$ and $B^{CB} > 0$ in the steady state, it follows that $\zeta^{CB} < 0$.

[37]In order to sustain a positive primary deficit in the steady state, $\zeta^G + \zeta^F - \zeta^T > 0$, through non-explosive government bond issuance, we require $R^G < \Pi \cdot GA \cdot GN$. This condition is satisfied under our calibration.

3.2 Results

3.2.1 Model Calibration

We utilize the publicly available data on (i) treasury yields, (ii) Federal Reserve's holdings of the treasury bonds spanning from December 2002 to June 2007, and (iii) U.S. Treasury's outstanding bonds from January 1990 to January 2007,[38,39] in calibrating the term-structure parameters of the model. We set $F = 120$ to account for maturities up to 30 years (i.e., 120 quarters). Parameter values are summarized in Table 1.2 in Appendix. Standard macroeconomic parameters are calibrated according to values widely accepted in the literature.

Recall that $z_{n,t}^f$ shock, governing the household's portfolio demand for maturity-f bonds, follows a Fréchet distribution with z_t^f and κ_B as scale and shape parameters, respectively. Similarly, $z_{m,t}^K$ affects the household's loan investment demand, and follows a Fréchet with z_t^K and κ_S as scale and shape parameters. Our calibration strategy is to use the slope of the steady state yield curve to calibrate $\{z_t^f\}_{f=1}^F$ and its level to calibrate z^K given (κ_B, κ_S).

Finally, we use $\gamma = 3$, which is an upper bound for the levels we observe for advanced economies' private debt to GDP ratio.

Shape parameters We use Fréchet shape parameters $\kappa_B = 10$, which we estimate based on the macro data with our bond portfolio equation (3) in Appendix B, and $\kappa_S = 6$ based on Kekre and Lenel (2023).[40] Using $\kappa_B = 10$ and $\kappa_S = 6$, the scale parameters $\{z^f\}_{f=1}^F$ are calibrated to match the yield curve's shape (i.e., relative yields across different maturities), while the scale parameter z^K is calibrated to match the model's steady-state return on the household's bond portfolio, R^{HB}. The precise calibration procedure for $\{z^f\}_{f=1}^F$ and z^K is outlined in Appendix B.1. Under our calibration, the level of untargeted R^K at the steady state is 8.12%, which is close to the average Moody's Seasoned BAA Corporate Bond Yield during 1990-2007, 7.88%.

Figure 1 displays the bond shares across maturities of households, government, and central bank, and the resulting yield curve, where the calibrated z^K and $\{z^f\}_{f=1}^F$ are reported in Table 1.1 and Figure 1.1. It is noteworthy that $z^1 = 1$ is particularly large compared to z^f for $f \geq 2$, as the shortest yield has historically been low relative to longer-term yields.

[38] When data on yields are missing, we employ interpolation to generate a smooth yield curve.

[39] https://fiscaldata.treasury.gov/datasets/monthly-statement-public-debt

[40] Kekre and Lenel (2023) express the bond convenience yield, defined as private safe rate minus the US Tbill rate, as a function of demand shocks and bond supplies. The convenience yield falls with the bond supply, with inverse elasticity as coefficient. In this environment, they use the Federal Reserve's announcement of the increase of dollar swap lines (e.g., 2-weeks period) to estimate the elasticity, which is 6.

This discrepancy may account for the safety and/or liquidity premium of short-term bonds extensively documented in the literature, including Krishnamurthy and Vissing-Jorgensen (2012) and Caballero and Farhi (2017).

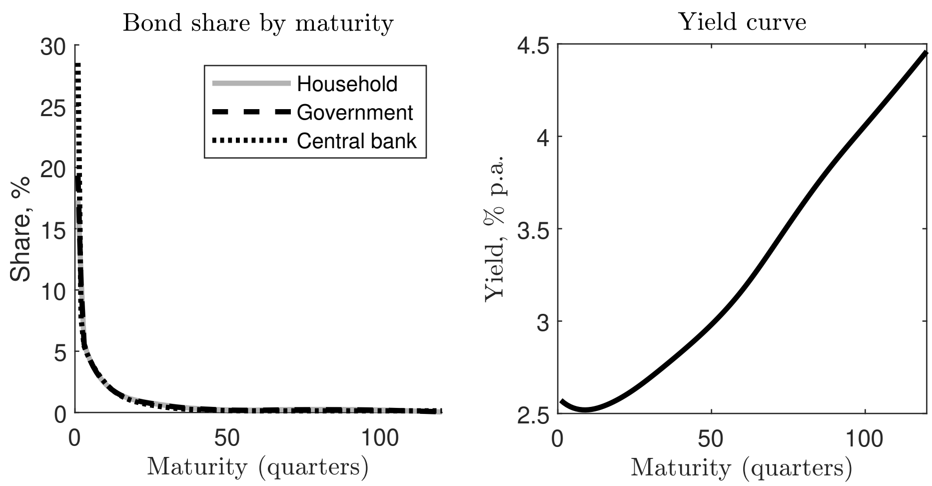


Figure 1: Steady-state bond shares of different entities, with the yield curve

3.2.2 Government's Supply and Central Bank's Demand for Bonds

We now study the impact of variations in the government's debt structure across maturities, captured by the government's Treasury issuance shares $\{\lambda^{G,f}\}_{f=1}^{F}$, on the steady-state yield curve. Figure 2 presents these variations,[41] with the left panel illustrating alternative debt issuance arrangements across maturities, and the right panel displaying the corresponding changes in the steady-state yield curve. The model generates a positive correlation between yields and the relative supply portfolio (i.e., $\lambda^{G,f}$), consistent with the literature.[42] A higher issuance of bonds of a given maturity raises the yield of those bonds. This effect not only pertains to the targeted bond maturity but also influences the overall equilibrium returns of both Treasury and private loan markets through the household's portfolio rebalancing. These changes also impact the government's gross bond issuance in general equilibrium.

[41] In Figure 2, dashed and dotted lines corresponds to higher issuance of long-term compared to the benchmark solid line. The shift in portfolio shares is arbitrary for the illustration purposes.

[42] Krishnamurthy and Vissing-Jorgensen (2012) find a higher debt-to-GDP ratio reduces the credit spreads, with the effect becoming more pronounced for longer maturities. Similarly, Greenwood and Vayanos (2014) find a positive correlation between the supply of long-term bonds relative to short-term bonds and the term spread, as illustrated in Figure 2.

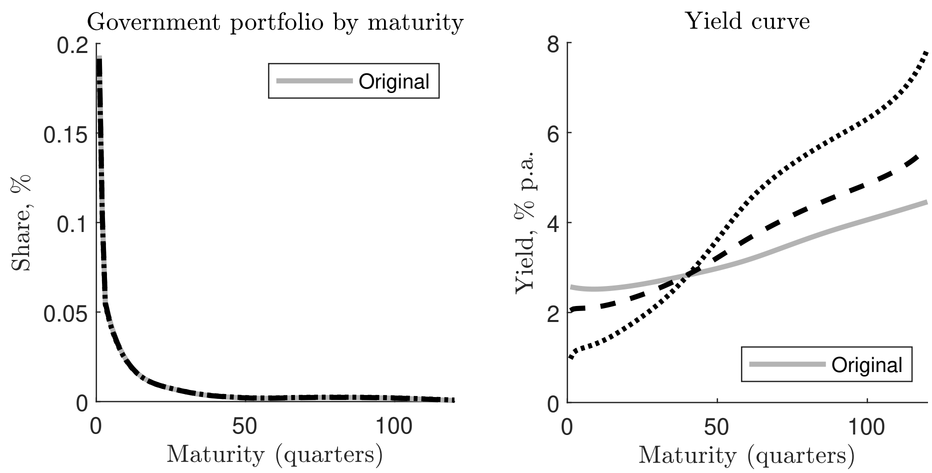


Figure 2: Government's bond issuance portfolio and yield curve

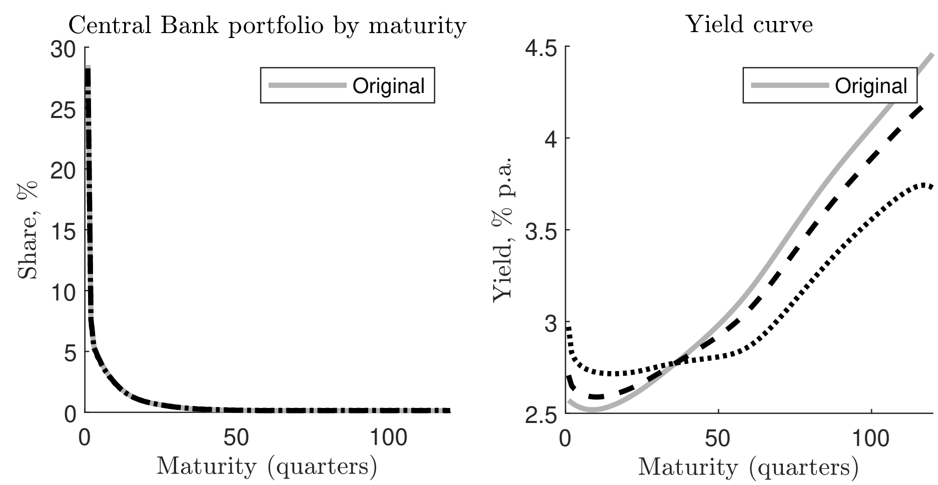


Figure 3: Variations in central bank's bond portfolio across maturities

Figure 3 depicts the alternative scenario in which the central bank modifies the composition of its bond portfolio.[43] The central bank's relative purchase of a given maturity is negatively correlated with its yield, consistent with the literature suggesting that the central bank's bond purchases can act as an additional monetary accommodation shock in segregated markets, e.g., Ray (2019) and Droste et al. (2021).[44] Our specification accommodates

[43]In Figure 3, dashed and dotted lines corresponds to higher purchases of long-term bonds compared with the benchmark solid line. The shift in portfolio shares is arbitrary for the illustration purposes.

[44]Krishnamurthy and Vissing-Jorgensen (2011) document that QE2, which primarily focused on treasury bonds, exerted a disproportionate impact on Treasuries and Agencies compared to mortgage-backed securities and corporate bonds. D'Amico and King (2013) identify stock and flow effects of QE programs on Treasury yields, supporting a view of imperfect substitution within the Treasury market.

integrated bond markets, wherein the household freely selects her bond portfolio, subject to allocation shocks distributed according to a Fréchet distribution, and generates a similar implication at the steady state.

3.2.3 Comparative Statics with Deficit Ratio

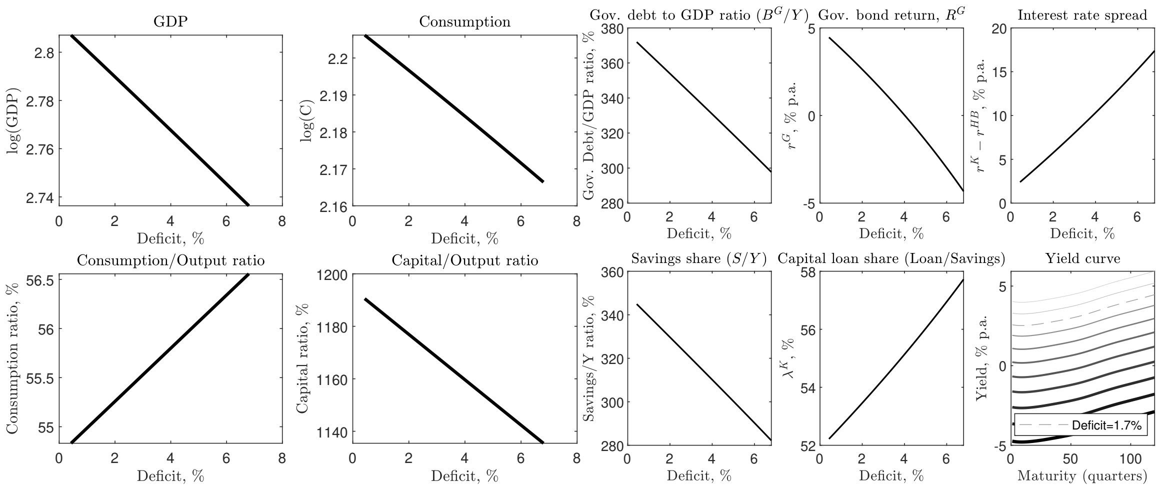


Figure 4: Variations in deficit ratio $\zeta^F + \zeta^G - \zeta^T$

Figures 4 present comparative statics with the deficit ratio $\zeta^F + \zeta^G - \zeta^T$. A higher deficit ratio can possibly be sustained only if the following conditions are satisfied: (i) the government issues more bonds, (ii) their effective bond rate R^G decreases, or (iii) output declines, thereby reducing total nominal deficit expenditure. We first examine the first case and observe that it is unsustainable in the long run: if the government issues more debt to finance a higher deficit (for a given output level), the government's effective return on bonds R^G increases (due to the supply effect described in Section 3.2.2), necessitating the issuance of even more debts to finance their additional interest costs. This process continues indefinitely, further pushing up R^G. It turns out that the second and third cases operate jointly: a higher deficit ratio reduces output, consumption, and capital, reducing the nominal deficit amount and the government's bond issuance, and depressing bond return R^G. The loan rate R^K remains relatively stable, and the credit spread $r^K - r^{HB}$ increases accordingly. Notably, our finding that the debt-to-GDP ratio $\frac{B^G}{Y}$ declines, while the entire yield curve shifts downward in response to an increase in the deficit ratio, aligns with previous literature.[45]

[45] Laubach (2009) empirically determined that a 1% point increase in the projected debt-to-GDP ratio is estimated to raise long-term interest rates by approximately 3-4 basis points.

We provide other relevant comparative statics results in Supplementary Material.

4 Short-Run Analysis

4.1 Log-linearization

We now provide the solution to the dynamic model under log-linear approximation. Lowercase letters represent normalized variables,[46] while hats represent log-deviations from the steady state levels. Owing to the complexity of the system, we discuss only a few key equilibrium equations in this section and delegate the comprehensive derivations and remaining equations to Online Appendix A.

Upon linearizing the Euler equation (i.e., equation (10)), the standard dynamic IS equation emerges, featuring the effective savings rate $\hat{r}^S_{t+1}$ of the household:

$$\hat{c}_t = \mathbb{E}_t\left[\hat{c}_{t+1} - \left(\hat{r}^S_{t+1} - \hat{\pi}_{t+1}\right)\right], \tag{37}$$

where $\hat{r}^S_t$ can be derived from equation (7) as

$$\hat{r}^S_t = \frac{\lambda^K\left(R^K - R^{HB}\right)}{R^S}\hat{\lambda}^K_{t-1} + \frac{(1-\lambda^K)R^{HB}}{R^S}\hat{r}^{HB}_t + \frac{\lambda^K R^K}{R^S}\hat{r}^K_t. \tag{38}$$

We observe that $\hat{r}^S_t$ depends on the household's effective bond rate $\hat{r}^{HB}_t$, the loan rate $\hat{r}^K_t$, and the share of savings channeled into firms as loans, $\hat{\lambda}^K_{t-1}$. The last $\hat{\lambda}^K_{t-1}$ term is capturing the portfolio relocation effect across asset classes (i.e., bonds and loans). Given $\hat{\lambda}^K_{t-1}$ is an endogenous variable dependent on the relative returns between bonds, $\hat{r}^{HB}_t$, and loans, $\hat{r}^K_t$, we characterize $\hat{r}^{HB}_t$ by linearizing (5) and obtain:

$$\hat{r}^{HB}_t = \sum_{f=1}^{F} \frac{\lambda^{HB,f}\left(YD^{f-1}\right)^{-(f-1)}}{R^{HB}\left(YD^f\right)^{-f}}\left[\hat{\lambda}^{HB,f}_{t-1} - (f-1)\cdot\hat{yd}^{f-1}_t + f\cdot\hat{yd}^f_{t-1}\right], \tag{39}$$

where $\hat{r}^{HB}_t$ relies on yields in the previous quarter $\{\hat{yd}^f_{t-1}\}^F_{f=1}$ as well as the current yields $\{\hat{yd}^{f-1}_t\}^F_{f=1}$, because the holding return of an f-maturity bond is determined by its quarter-to-quarter price change and, equivalently, yields. The term $\hat{r}^{HB}_t$ in equation (39) also depends on $\{\hat{\lambda}^{HB,f}_{t-1}\}^F_{f=1}$, which are the shares of household bond savings allocated to each

[46] For example, we define $k_t \equiv \frac{K_t}{A_{t-1}N_{t-1}}$, $y_t \equiv \frac{Y_t}{A_t N_t}$, $c_t \equiv \frac{C_t}{A_t N_t}$, $n_t \equiv \frac{N_t}{N_t}$, $p^K_t \equiv \frac{P^K_t}{P_t}$.

bond maturity, and capture the effect of endogenous portfolio relocation across maturities. To further examine the determinants of portfolio relocation, we linearize the expression for the household's optimal bond portfolio (i.e., (3)) and express the bond shares $\{\hat{\lambda}_{t-1}^{HB,f}\}_{f=1}^{F}$ as functions of past $\{\hat{yd}_{t-1}^{f}\}_{f=1}^{F}$ and current $\{\hat{yd}_{t}^{f-1}\}_{f=1}^{F}$ yields. Formally, this can be written as:

$$\hat{\lambda}_{t-1}^{HB,f} = \kappa^{B}\mathbb{E}_{t-1}\left[\hat{z}_{t-1}^{f} - \hat{\pi}_t + \hat{c}_{t-1} - \hat{c}_t - (f-1)\cdot\hat{yd}_t^{f-1} + f\cdot\hat{yd}_{t-1}^{f} - \hat{\phi}_{t-1}^{B}\right], \quad (40)$$

where $\hat{\phi}_t^B$ contains $\{\hat{yd}_t^{f-1},\ \hat{yd}_{t-1}^{f}\}_{f=1}^{F}$, along with other aggregate variables. By substituting the expression in (40) into (39), we can represent the household's effective bond rate as a function of yields along the term structure.

The relations between $\hat{r}_t^K$ and λ_t^K is characterized by linearizing the household's optimal portfolio between the bond and the loan markets (i.e., (6)), which is expressed as

$$\hat{\lambda}_t^K = \kappa^S\left(1-\lambda^K\right)\left(\hat{z}_t^K + \mathbb{E}_t\left[\hat{r}_{t+1}^K - \hat{r}_{t+1}^{HB}\right]\right),$$

where rises in $\hat{z}_t^K$ and the expected spread between the loan and the bond returns $\mathbb{E}_t[\hat{r}_{t+1}^K - \hat{r}_{t+1}^{HB}]$ raise the share $\hat{\lambda}_t^K$ of savings allocated to firms as loans. From (38), it can be noted that $\hat{r}_{t+1}^K$ directly influences the effective savings rate $\hat{r}_{t+1}^S$, thereby altering the consumption dynamics through the households' intertemporal substitution channel (i.e., (37)). Additionally, a change in $\hat{r}_{t+1}^K$ leads to a change in λ_t^K, which affects the loan issuance L_t, and thus the output from (30), which in turn impacts aggregate labor (i.e., (27)), and the capital (i.e., (29)) aggregation.

4.2 Welfare

In order to compare welfare across various policy regimes, we follow the previous literature (e.g., Woodford (2003) and Coibion et al. (2012)) and calculate a second-order approximation to the household utility function, which we summarize in Proposition 1.

Proposition 1 *A 2^{nd}-order approximation to the expected per-period welfare of the household is given by*

$$\mathbb{E}U_t - \bar{U}^F = \Omega_0 + \Omega_n \mathrm{Var}(\hat{n}_t) + \Omega_\pi \mathrm{Var}(\hat{\pi}_t) + \textit{t.i.p} + \textit{h.o.t},$$

where Ω_0, Ω_n, and Ω_π are provided in (C.38) in Online Appendix C, and $\bar{U}^F$ represents the *efficient (flexible-price) steady-state utility of the household, around which our approximation is centered.*[47]

The term $\Omega_0 < 0$ arises under our positive steady-state inflation due to a first-order welfare loss at the steady-state allocation relative to the efficient (i.e., flexible-price) steady-state. Given that we have positive trend inflation, this term can be incorporated into the t.i.p. The coefficients $\Omega_n > 0$ and $\Omega_\pi > 0$ associated with the variance terms of labor and inflation gaps, respectively, capture the disutility of business cycle fluctuations.

This welfare characterization proves valuable for conducting comparisons across multiple monetary policy regimes: (i) conventional policy, (ii) yield-curve-control policy, and (iii) mixed policy. The mixed policy regime is characterized by the central bank implementing the conventional policy outside the zero lower bound (ZLB), and yield-curve-control when the ZLB binds. We further consider this last case as it more closely mirrors the policy approach adopted by the majority of central banks in practice.

4.3 Results

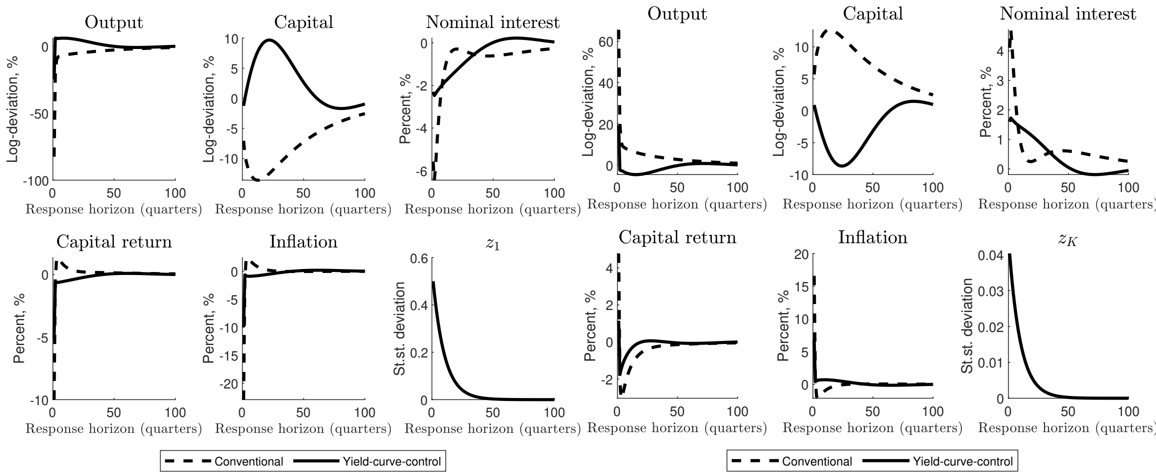
4.3.1 Impulse-Response without the ZLB

First, we analyze the impulse-responses to various shocks when the economy does not enter the ZLB. The shocks considered include z_t^1 and z_t^K,[48] representing household preferences for bonds and loans, respectively, and ε_t^T (i.e.,fiscal or tax shock). Graphs for other shocks are included in Supplementary Material.

Short-term bond preference shock, z_t^1: Figure 5a showcases the impulse-responses to the z_t^1 shock, which influences the household's portfolio demand for the bond of the shortest maturity. The dashed lines illustrate the impulse responses under the conventional monetary policy, while the solid lines depict those under the yield-curve-control regime.

[47] The acronyms t.i.p. and h.o.t. denote terms independent of policy and higher-order terms, respectively.

[48] For z_t^1 and z_t^K shocks, we express the magnitude of initial shocks in terms of deviation from the steady state values, instead of multiple of their standard deviations, as those standard deviations are set to be small.



(a) z_t^1 shock

(b) z_t^K shock

Figure 5: Impulse response to z_t^1 and z_t^K without ZLB

In a conventional policy setting, an increase in z_t^1 is associated with a higher household portfolio demand for short-maturity bonds. This results in a decline in the short rate, which subsequently reduces returns on bonds of other maturities and loans, as well as the wage.[49] As households re-optimize their portfolio choices and firms substitute between capital and labor, inflation drops. This leads to a drop in output as the labor supply diminishes due to the declining wage. Although the initial monetary policy response aims to boost aggregate demand via consumption and investment, it is insufficient to prevent a fall in output. Under our calibration, a one standard deviation increase in z_t^1 reduces output by $3-4\%$.

The yield-curve-control policy effectively insulates the economy from a z_t^1 shock. The rationale is straightforward: as z_t^1 shocks affect the economy primarily by distorting households' bond-portfolio decisions in the segmented asset markets (as captured by $\kappa_B, \kappa_S < \infty$), the central bank can mitigate this distortion by adjusting its own bond portfolios. With a positive z_t^1 shock, the central bank prevents the yield curve from shifting downwards by exerting upward pressure on the effective bond market return of households. This intervention raises both the loan rates and wages (i.e., factor prices), and inflation remains stable as a result. Consequently, labor supply and output experience minimal changes.

In this specific context, a positive z_t^1 shock can be interpreted more broadly as a special case of bond market disruptions, such as a rise in the degree of flight to safety or liquidity, as

[49]That the household's endogenous portfolio choice is now a function of relative rates of different assets is crucial for generating this phenomenon. For instance, if the household's portfolio is fixed, a positive z_t^1 shock would cause the loan rate to rise as the household's loan investment decreases.

the shortest-maturity bond (e.g., federal fund market) features the highest degrees of safety and liquidity. Under the conventional policy, this shock generates an endogenous recession, while a yield-curve-control regime allows the central bank to actively manipulate the entire term structure and achieve near-perfect stabilization.

Loan preference shock, z_t^K: Figure 5b displays the impulse-responses to the z_t^K shock. A positive shock in z_t^K prompts the household to invest more to the firms via loans, reducing the capital return and increasing aggregate capital. Consequently, output and inflation rise, with the central bank raising the policy rate. As previously observed, a yield-curve-control regime proves to be more effective for stabilization.

Tax Shock, ε_t^T: Figure 6 displays the impulse-responses to a ε_t^T shock, which increases the government's tax revenues. Under the conventional policy, an upward shift in ε_t^T leads to a reduced issuance of risk-free bonds by the government. This causes a decline in bond and loan returns, and factor prices (i.e., wages, and inflation), through the same channels of the household's endogenous portfolio reallocation and firms' input substitution as previously discussed. Under our calibration, the conventional monetary policy response is not sufficient in counteracting the negative effects of a decrease in bond issuance on output.[50]

The yield-curve-control policy achieves better stabilization with a significantly smaller movement in the short-term yield. As the entire yield curve shifts downward in response to the shock, smaller adjustments of each individual maturity are required to attain a similar reduction in the effective household savings rate r_{t+1}^S, which boosts aggregate consumption and mitigates the negative impacts of the lower bond issuance.

4.3.2 Impulse-Response at the ZLB

In this section, we display the impulse-responses to various shocks when monetary policy is constrained at the ZLB. To enable each structural shock to independently push the economy into the ZLB from its initial steady-state (and for enhanced graphical representation), we calibrate the size of the shocks to very large levels. It is important to note that, in reality, shocks of this magnitude are highly improbable.

[50]Note that our model is not Ricadian: due to the segmented markets of bonds and loans, we have a direct effect of fiscal shocks on the business cycle throught their impacts on the government's bond issuance.

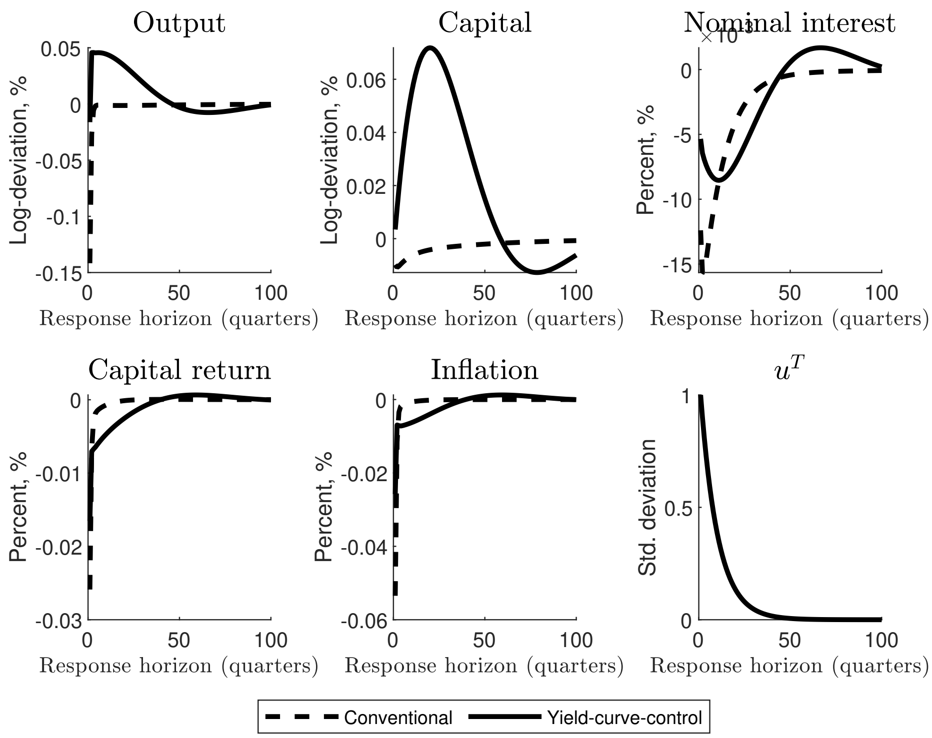
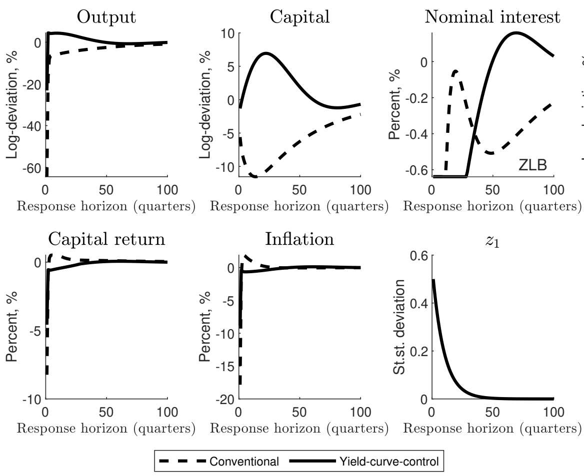


Figure 6: Impulse-response to ε_t^T shock without ZLB

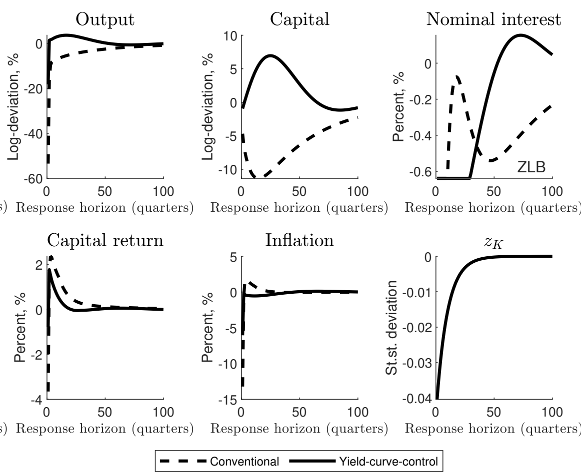
Short-term bond preference shock, z_t^1: Figure 7a displays the impulse-responses to a z_t^1 shock that influences the household's portfolio demand for the shortest maturity bond. The dashed lines illustrate the impulse-responses under the conventional monetary policy framework, while the solid lines depict the impulse-responses under the yield-curve-control setting. Figure 7a exhibits similar behavior to Figure 5a (i.e., the case without ZLB), with the exception of the short-term rate being constrained at the ZLB for several quarters.

Yield-curve-controls achieve nearly perfect stabilization, as demonstrated in Figure 5a. However, it is important to note that this policy results in a more extended ZLB duration compared to the conventional policy. When the economy enters a ZLB episode under the yield-curve-control regime, the central bank increases its purchase of long-term bonds, consequently lowering long-term yields. This action exerts additional downward pressures on short-term yields and the private loan rates in the household's portfolio problem, causing the ZLB constraint to bind for a longer duration. In our calibration, this endogenous portfolio effect outweighs the additional stabilization offered by the yield-curve-control regime, which (for a fixed portfolio allocation) would tend to facilitate the economy's exit from the ZLB sooner.

In sum, although a yield-curve-control policy aids in insulating the economy from various shocks, it generates extended ZLB episodes when the short-term rate becomes constrained. As a result, unconventional policies gain increased importance for economic stabilization, and the central bank relies more heavily on them to mitigate the adverse effects of additional shocks.[51]



(a) z_t^1 shock



(b) z_t^K shock

Figure 7: Impulse response to z_t^1 and z_t^K with ZLB

Loan preference shock, z_t^K: Figure 7b depicts the impulse-responses to a z_t^K shock at the ZLB. A sizable negative shock to z_t^K induces the households to invest less in private loans, and rebalance their portfolio toward bond markets. Consequently, bond rates decline, and the policy rate becomes constrained by the ZLB. Output, capital, inflation, and the loan rates all drop in response. The yield-curve-control policy effectively stabilizes the economy while generating a longer ZLB episode in a similar way to Figure 7a.

Tax shock, ε_t^T: Figure 8 depicts the impulse-responses to a positive ε_t^T shock, resulting in increased tax revenues. As in Figure 6, the economy experiences a recession under the conventional policy. Following a positive tax shock, the government significantly reduces its bond issuance, driving the economy into a ZLB recession. In turn, output, capital, inflation, and capital returns all decline in response. This experiment underscores the stabilizing role of the supply of safe bonds at the ZLB, when markets are segmented so that those safe

[51] In Karadi and Nakov (2021), quantitative easing (QE) policies effectively stabilize financial disruptions within the banking system, even if this comes at the cost of banks becoming increasingly reliant on the power of QEs. Despite the absence of explicit roles of banks in our model, it exhibits the similar characteristics.

bonds play special roles, consistent with previous findings including Caballero and Farhi (2017) and Caballero et al. (2021). Under the yield-curve-control regime, the central bank

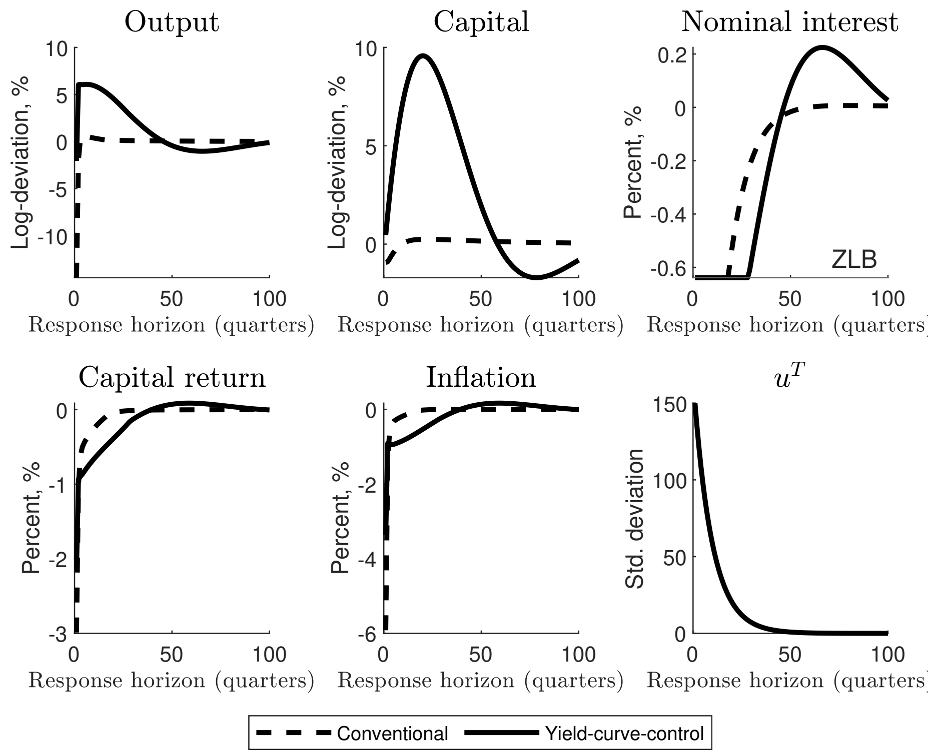


Figure 8: Impulse-response to ε_t^T shock with ZLB

lowers the entire yield curve and reduces the household's effective savings rate. This action stimulates aggregate demand, causing output and capital to increase in response. Inflation and capital returns decrease less than in the conventional policy case. It is important to note that, in this scenario as well, yield-curve-control generates a longer ZLB episode compared to conventional policy, primarily due to the endogenous portfolio choices of households.

4.3.3 Policy Comparison

Based on the welfare criterion in Proposition 1, we now compare various monetary policy regimes. As detailed in Section 4.2, we examine the following policy frameworks: (i) **conventional policy** (i.e., (23)), (ii) **yield-curve-control policy** (i.e., (24)), and (iii) **mixed policy**, wherein the central bank implements yield-curve-control exclusively when the policy rate reaches the ZLB.[52] In the context of these three distinct monetary policy regimes,

[52] Upon the economy's departure from the ZLB, the central bank instantly changes its holdings of $f \geq 2$ bonds to its steady-state holding levels. This is different from the approach of Karadi and Nakov (2021), who

we compute the following metrics: (i) ex-ante per-period welfare, (ii) mean and median ZLB duration, and (iii) frequency with which the economy resides at the ZLB.

	Conventional	**Yield-Curve-Control**	**Mixed Policy**
Mean ZLB duration	4.5533 quarters	6.2103 quarters	5.5974 quarters
Median ZLB duration	3 quarters	3 quarters	2 quarters
ZLB frequency	15.9596%	13.4242%	17.4141%
Welfare	-1.393%	-1.2424%	-1.3662%

Table 1: Policy comparisons

The results presented in Table 1 can be summarized as follows: (i) in comparison to the conventional policy, both yield-curve-control and mixed policies enhance welfare by 0.16 and 0.03 percentage points, respectively; (ii) the yield-curve-control policy extends ZLB episodes, exhibiting a longer spell duration (i.e., 6.2 quarters) relative to the conventional policy (i.e., 4.6 quarters), and (iii) the mixed policy attains the ZLB spell and welfare, both between levels implied by the conventional and yield-curve-control policies.

The mixed policy has better stabilization at the ZLB than the conventional policy, as it employs the yield-curve-control policy during the ZLB. While this effect can lower the ZLB duration compared to the conventional policy, it might extend the ZLB spell as well[53] from the household's portfolio rebalancing channel as detailed in Section 4.3.2. In contrast to the mixed policy, yield-curve-controls enable the central bank to manipulate the entire yield curve even beyond ZLB periods. When confronted with adverse shocks, yield-curve-control reduces long rates even prior to the economy's entry into the ZLB, propping up the aggregate demand and thereby lowering the probability to actually enter the ZLB, i.e., ZLB frequency. However, it imposes strongest downward pressures on the short rates, leading to the longest average ZLB spell.[54]

show the optimality of a gradual exit from quantitative easing (QE) policies, as banks in their model possess diminished incentives to recapitalize without supplementary QE policies.

[53] As we observe, the mixed policy generates higher mean ZLB duration and lower median ZLB duration, compared with the conventional policy.

[54] Note that those statistics in Table 1 are ex-ante, i.e., all kinds of shocks can potentially hit the economy. For each specific shock, ZLB duration and frequency might feature different shock-dependent patterns under different policy regimes.

5 Conclusion

This paper presents a New-Keynesian model that integrates the term structure of financial markets, as well as the size and maturity structure of the government's and central bank's balance sheets, as active elements that influence the business cycle. We demonstrate that the market segmentation across asset classes and maturities, in conjunction with the household's endogenous portfolio reallocation channel, are two essential components for comprehending the functioning of unconventional monetary policy. To this end, we develop a model in which the asset market segmentation arises due to incomplete information about asset returns, resulting in an equilibrium term structure that deviates from the expectation hypothesis usually found in standard models.

We indicate that the government's issuance and the central bank's purchase of different bond maturities serve as two primary determinants of the yield curve's level and slope. Additionally, the government's issuance of risk-free bonds stimulates the economy when conventional monetary policy is constrained by the ZLB, as documented in previous research on the "safe-asset shortage problems". We also examine various monetary policy regimes, revealing that yield-curve-control (YCC) interventions —where the central bank actively manipulates the entire yield curve— provide greater stabilization than conventional policy, both during normal periods and ZLB episodes. However, YCC policy exhibits intriguing side effects, as it increases the duration of ZLB episodes. This outcome stems from the portfolio balancing channel: by easing long-term rates, the central bank indirectly applies additional downward pressure on short-term rates by inducing households to endogenously rebalance their portfolios towards shorter maturities. Consequently, unconventional policies become *addictive* in the end: central banks rely on them as the most potent tools at the ZLB, yet simultaneously perpetuate the ZLB conditions that render the conventional policy ineffective.

We believe that our model will be valuable for future research investigating the effects of quantitative tightening policies resulting from recent episodes of high inflation. Furthermore, our model is well-suited for studying the political economy implications and risks to taxpayers arising from the expansion of the central bank's balance sheet.[55] Lastly, we aim to extend our framework to the international macro literature, revisiting the topics of global imbalance issues (e.g., Caballero et al. (2008, 2021)) and monetary cycles (e.g., Miranda-

[55]For instance, Karadi and Nakov (2021) introduced a small quadratic efficiency cost to QE policies as a reduced-form proxy for un-modeled distortions and political costs associated with maintaining a positive central bank balance sheet.

Agrippino and Rey (2021)) with the incorporation of endogenous fluctuations in the term structure of interest rates.

References

Ang, Andrew and Monika Piazzesi, "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables," *Journal of Monetary Economics*, 2003, *50*, 745–787.

Angeletos, George-Marios and Jennifer La'O, "Sentiments," *Econometrica*, 2013, *81* (2), 739–779.

Bekaert, Geert, Seonghoon Cho, and Antonio Moreno, "New Keynesian Macroeconomics and the Term Structure," *Journal of Money, Credit and Banking*, 2010, *42* (1), 33–62.

Caballero, Ricardo J. and Emmanuel Farhi, "The Safety Trap," *Review of Economic Studies*, 2017, *85* (1), 223–274.

_ , _ , and Pierre-Olivier Gourinchas, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, March 2008, *98* (1), 358–93.

_ , _ , and _ , "Global Imbalances and Policy Wars at the Zero Lower Bound," *Review of Economic Studies*, 2021.

Calvo, Guillermo, "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Christensen, Jens H E. and Signe Krogstrup, "A Portfolio Model of Quantitative Easing," *Working Paper*, 2018.

_ and _ , "Transmission of Quantitative Easing: The Role of Central Bank Reserves," *The Economic Journal*, 2019, *129* (617), 249–272.

Coibion, Olivier, Yuriy Gorodnichenko, and Johannes Wieland, "The optimal inflation rate in New Keynesian models," *Review of Economic Studies*, 2012, *79*, 1371–1406.

Cúrdia, Vasco and Michael Woodford, "The central-bank balance sheet as an instrument of monetary policy," *Journal of Monetary Economics*, 2011, *58*, 54–79.

D'Amico, Stefania and Thomas B. King, "Flow and stock effects of large-scale treasury purchases:Evidence on the importance of local supply," *Journal of Financial Economics*, 2013, *108* (2), 425–448.

Dordal i Carreras, Marc, Matthias Hoelzlein, and Jens Orben, "Efficiency, Risk and the Gains from Trade in Interbank Markets," *Job Market Paper*, 2023.

Droste, Michael, Yuriy Gorodnichenko, and Walker Ray, "Unbundling Quantitative Easing: Taking a Cue from Treasury Auctions," *Working Paper*, 2021.

Duffie, Darrell and R. Kan, "A yield-factor model of interest rates," *Mathematical Finance*, 1996, *6*, 379–406.

Eaton, Jonathan and Samuel Kortum, "Technology, geography, and trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Ennis, Huberto M., "A simple general equilibrium model of large excess reserves," *Journal of Monetary Economics*, 2018, *98*, 50–65.

Froot, Kenneth A., "New Hope for the Expectation Hypothesis of the Term Structure of Interest Rates," *The Journal of Finance*, 1989, *44*, 283–305.

Frost, Peter A., "Banks' Demand for Excess Reserves," *Journal of Political Economy*, 1971, *79* (4), 805–825.

Gertler, Mark and Peter Karadi, "A model of unconventional monetary policy," *Journal of Monetary Economics*, 2011, *58*, 17–34.

Gourinchas, Pierre-Olivier, Dimitri Vayanos, and Walker Ray, "A Preferred-Habitat Model of Term Premia, Exchange Rates, and Monetary Policy Spillovers," *Working Paper*, 2022.

Greenwood, Robin and Dimitri Vayanos, "Bond Supply and Excess Bond Returns," *The Review of Financial Studies*, 2014, *27* (3), 663–713.

_, Samuel G. Hanson, Jeremy C. Stein, and Adi Sunderam, "A Quantity-Driven Theory of Term Premia and Exchange Rates," *Working Paper*, 2022.

Gumbel, E.J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Güntner, Jochen H.F., "The federal funds market, excess reserves, and unconventional monetary policy," *Journal of Economic Dynamics & Control*, 2015, *53*, 225–250.

Humpage, Owen F, "The Fed's yield-curve-control policy," *Economic Commentary*, 2016, *1* (2016-15).

Ireland, Peter, "Technology Shocks in the New Keynesian Model," *Review of Economics and Statistics*, 2004, *86* (4).

Karadi, Peter and Anton Nakov, "Effectiveness and addictiveness of quantitative easing," *Journal of Monetary Economics*, 2021, *117*, 1096–1117.

Kekre, Rohan and Moritz Lenel, "The Flight to Safety and International Risk Sharing," *Working Paper*, 2023.

_ , _ , and Federico Mainardi, "Monetary Policy, Segmentation, and the Term Structure," *Working Paper*, 2023.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," *Brookings Papers on Economic Activity*, 2011, *1* (22).

_ and _ , "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 2012, *120* (2), 233–267.

Laubach, Thomas, "New Evidence on the Interest Rate Effects of Budget Deficits and Debt," *Journal of the European Economic Association*, 2009, *7* (4), 858–885.

Mattingly, Mary and Ahmed S. Abou-Zaid, "The High Levels of Excess Reserves 2008-2012: An Investigation into the Determinants of the U.S. Banks' Liquidity Hoarding during the Global Financial Crisis," *Advances in Economics and Business*, 2015, *3* (4), 141–148.

Miranda-Agrippino, Silvia. and Hélène Rey, "The Global Financial Cycle," *Handbook of International Economics*, 2021, *Forthcoming.*

Modigliani, Franco and Richard Sutch, "Innovations in Interest Rate Policy," *American Economic Review*, 1966, *56* (1/2), 178–197.

Primus, Keyra, "Excess reserves, monetary policy and financial volatility," *Journal of Banking and Finance*, 2017, *74*, 153–168.

Ray, Walker, "Monetary Policy and the Limits to Arbitrage: Insights from a New Keynesian Preferred Habitat Model," *Working Paper*, 2019.

Rudebusch, Glenn D. and Tao Wu, "A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy," *The Economic Journal*, 2008, *118*, 906–926.

Sims, Eric and Cynthia Wu, "Evaluating Central Banks' Tool Kit: Past, Present, and Future," *Journal of Monetary Economics*, 2021, *118*, 135–160.

Swanson, Eric T. and John C. Williams, "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates," *American Economic Review*, 2014, *104* (10), 3154–3185.

Vayanos, Dimitri and Jean-Luc Vila, "A Preferred-Habitat Model of the Term Structure of Interest Rates," *Econometrica*, 2021, *89* (1), 77–112.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.

Appendix Tables and Figures

1 Calibration

Calibrated steady-state parameters		
$\{z^f\}_{f=1}^F$	See Figure 1.1	Bond maturity scale (mean) parameters
z^K	1.0089	Capital scale (mean) parameter
$\frac{C}{A\bar{N}}$	9.0112	Normalized consumption
$\frac{Y}{A\bar{N}}$	16.3315	Normalized output
$\frac{K}{A\bar{N}}$	192.6312	Normalized capital
$\frac{C}{Y}$	0.5518	Consumption per GDP
$\frac{K}{Y}$	11.7951	Capital per GDP
$\frac{P^K}{P}$	0.0335	Normalized rental price of capital
$\lambda^{HB,f}$	See Figure 1	Household's bond portfolio
λ^K	0.5322	Household's loan share out of total savings
R^K	1.0203	Household's loan rates (quarterly)
YD^f	See Figure 1	Equilibrium yield curve

Table 1.1: Steady-state values with parameters in Table 1.2

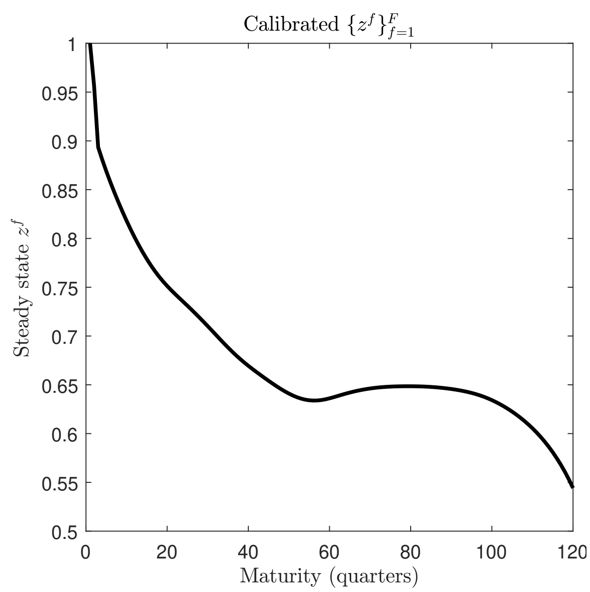


Figure 1.1: Calibrated scale parameters of the Fréchet distribution, $\{z^f\}_{f=1}^F$

Households		
(β, η)	$(0.998, 1)$	Discount factor and Frisch labor elasticity
GN	1.00275	Population growth rate
Intermediate good firms		
μ	0.00375	Technology growth rate
γ_L	3	Loan-to-GDP ratio
α	0.4	Capital income share
ϵ	10	Elasticity of substitution between differentiated goods
θ	0.45	Calvo price stickiness parameter
σ_A	0.00225	Standard deviation of technology shock
δ	0.025	Capital depreciation rate
Term structure		
κ_S	6	Capital shape parameter: Kekre and Lenel (2023)
(ρ_z, ρ_z^K)	$(0.9, 0.9)$	Autoregressive coefficient: z_t^f and z_t^K
(σ_z, σ_z^K)	10^{-8}	Standard deviation: z_t^f and z_t^K
Government		
ζ^F	0.1111	Government's optimal subsidy to firms
ζ^G	0.0789	Government expenditure per GDP
a^G	11.6761	Government expenditure coefficient
$\zeta^F + \zeta^G - \zeta^T$	0.017	Government deficit per GDP
ζ^T	0.1730	Government tax revenue per GDP
(ρ_G, ρ_T)	$(0.9, 0.9)$	Autoregressive coefficient: expenditure and tax shocks
(σ_G, σ_T)	0.00148	Standard deviation: expenditure and tax shocks
Central bank		
ζ^{CB}	-0.18	Central bank's balance sheet per issued government bond
$\bar{\pi}$	$\frac{0.023}{4} = 0.00575$	Trend inflation (steady-state inflation)
γ_π^1	2.5	Taylor coefficient of YD_t^1: responsiveness to inflation
$\gamma_\pi^{f\geq 2}$	1.5	Taylor coefficient of $YD_t^{f\geq 2}$: responsiveness to inflation
γ_y	0.15	Taylor coefficient: responsiveness to output
$\gamma_y^{f\geq 2}$	0.15	Taylor coefficient of $YD_t^{f\geq 2}$: responsiveness to output
(ρ_1, ρ_2)	$(1.05, -0.13)$	Autocorrelation in monetary policy: Coibion et al. (2012)
σ^{YD^1}	0.0006	Standard deviation: monetary policy shock (for YD_t^1)
$\sigma^{YD^{f\geq 2}}$	4×10^{-9}	Standard deviation: monetary policy shock (for $YD_t^{f\geq 2}$)
τ^{YD}	$I_{F\times F}$	State reduction matrix (for $YD_t^{f\geq 2}$)

Table 1.2: Parameter values

2 Additional Figures

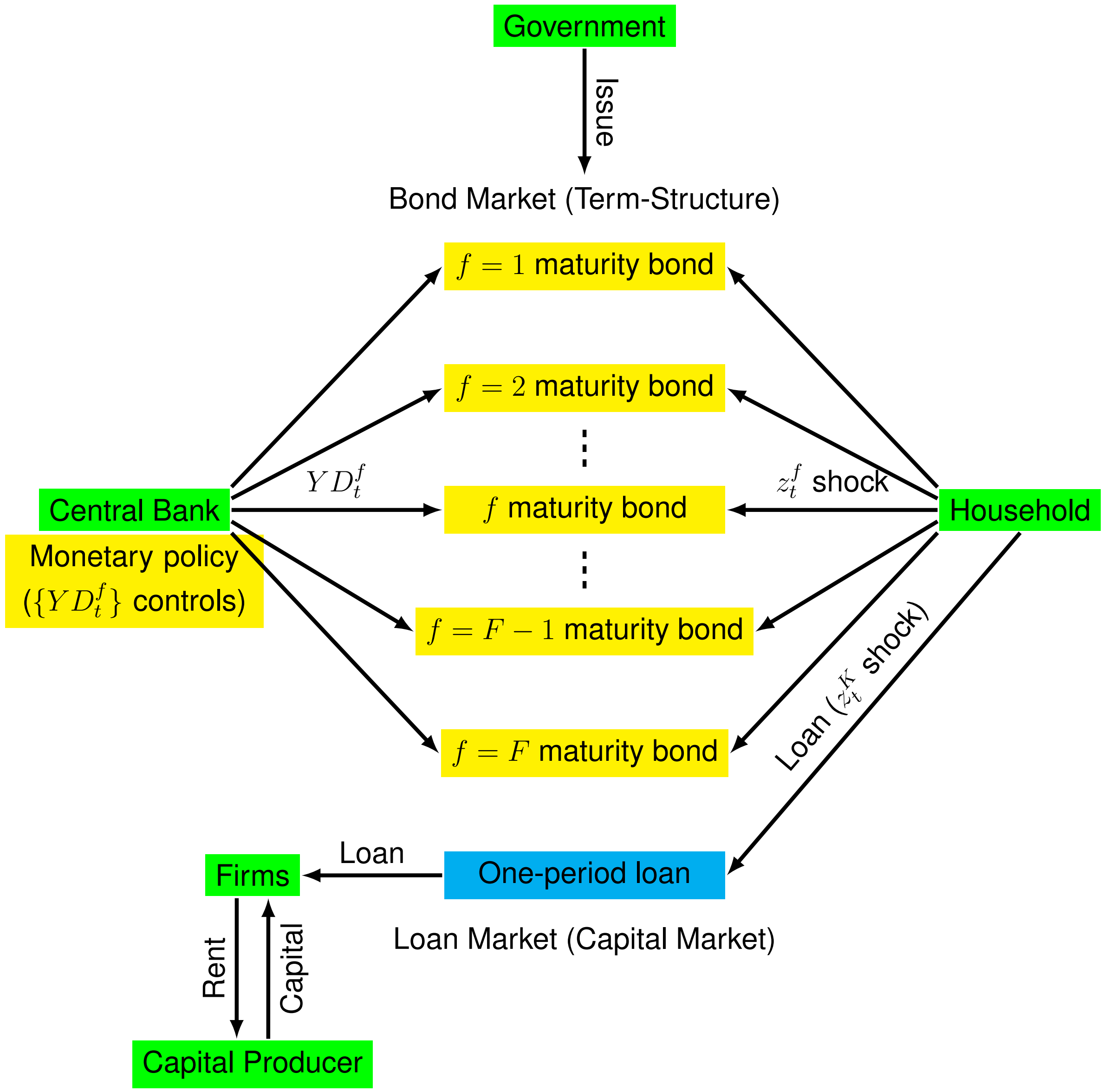


Figure 1.2: Markets, Agents, and Mechanisms: households allocate wealth between bonds and extending loans to intermediate goods producers. The government issues bonds with $f = 1 \sim F$ maturities. Under a conventional monetary policy, the central bank manipulates the yield of $f = 1$ bond without adjusting the holdings for longer-term bonds. Under the yield-curve-control, the central bank manages yields of $f = 1 \sim F$.

Online Appendix: For Online Publication Only

Appendix A Derivation and Proofs

A.1 Detailed Derivations in Section 2

As in (15), an intermediate firm ν resetting its price at period t maximizes

$$\max \sum_{j=0}^{\infty} \mathbb{E}_t \left[\theta^j Q_{t,t+j} \cdot \left[\left[1 - \gamma_L \cdot \left(\widetilde{R}^K_{t+j+1} - 1 \right) \right] \cdot (1+\zeta^F) \cdot P_{t+j}(\nu) Y_{t+j}(\nu) \right.\right. \\ \left.\left. - W_{t+j}(\nu) N_{t+j}(\nu) - P^K_{t+j-1} K_{t+j-1}(\nu) \right] \right], \tag{A.1}$$

where $Q_{t,t+j}$ is the firm's stochastic discount factor between periods t and $t+j$ and ζ^F is a production subsidy. Also, we define $\widetilde{R}^K_{t+j+1} \equiv \mathbb{E}_{t+j}[Q_{t+j,t+j+1} R^K_{t+j+1}]$. Solving for the optimal resetting price at period t, P^*_t, we obtain

$$\frac{P^*_t}{P_t} = \frac{\mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\epsilon+1} Y_{t+j} \left(\frac{(1+\zeta^F)^{-1}\epsilon}{\epsilon - 1} \right) \left(\frac{MC_{t+j|t}(\nu)}{P_{t+j}} \right) \right]}{\mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\epsilon} Y_{t+j} \left[1 - \gamma_L \cdot \left(\widetilde{R}^K_{t+j+1} - 1 \right) \right] \right]}, \tag{A.2}$$

where subindex $t+j|t$ represents the value of the variable conditional on the firm having reset its price last time at period t, and $MC_{t+j|t}(\nu)/P_t$ is the real marginal cost of production, defined as

$$\frac{MC_{t+j|t}(\nu)}{P_{t+j}} = \left(\frac{P^K_{t+j}}{P_{t+j}} \right)^{\alpha} \left(\frac{W_{t+j|t}(\nu)}{P_{t+j} A_{t+j}} \right)^{1-\alpha}. \tag{A.3}$$

A.1.1 Detailed Derivation in Section 2.9

Using equations equation (9), equation (12), equation (13) and equation (A.3) we can express firm-specific marginal costs as a function of the aggregate variables as in

$$\frac{MC_{t+j|t}(\nu)}{P_{t+j}} = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{C_{t+j}}{A_{t+j}\bar{N}_{t+j}} \right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_{t+j}}{A_{t+j}\bar{N}_{t+j}} \right)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{P^K_{t+j}}{P_{t+j}} \right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{P^*_t}{P_{t+j}} \right)^{-\left(\frac{\epsilon(1-\alpha)}{\eta+\alpha}\right)}.$$

Similarly, we integrate loan and labor demand across the continuum of firms and obtain the following expressions for the loan and labor aggregation conditions.

$$\frac{K_t}{A_{t-1}\bar{N}_{t-1}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \cdot GA_t \cdot GN \cdot \left(\frac{C_t}{A_t\bar{N}_t}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\frac{P_t^K}{P_t}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \Delta_t, \tag{A.4}$$

$$\frac{N_t}{\bar{N}_t} = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{P_t^K}{P_t}\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}}, \tag{A.5}$$

where Δ_t is a measure of price-dispersion that can be recursively defined as

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\Delta_{t-1}. \tag{A.6}$$

Plugging the real marginal cost and the expressions for Q_{t+j} into the optimal resetting price equation (i.e., equation (A.2)), we obtain

$$\left(\frac{P_t^*}{P_t}\right)^{1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)} = \frac{\mathbb{E}_t\left[\sum_{j=0}^{\infty}(\theta\beta)^j (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\left(\frac{C_{t+j}}{A_{t+j}\bar{N}_{t+j}}\right)^{-\alpha\frac{\eta+1}{\eta+\alpha}}\left(\frac{Y_{t+j}}{A_{t+j}\bar{N}_{t+j}}\right)^{\frac{\eta+1}{\eta+\alpha}}\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon\frac{\eta+1}{\eta+\alpha}}\left(\frac{P_{t+j}^K}{P_{t+j}}\right)^{\alpha\frac{\eta+1}{\eta+\alpha}}\right]}{\mathbb{E}_t\left[\sum_{j=0}^{\infty}(\theta\beta)^j\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon-1}\left(\frac{C_{t+j}}{A_{t+j}\bar{N}_{t+j}}\right)^{-1}\left(\frac{Y_{t+j}}{A_{t+j}\bar{N}_{t+j}}\right)\left[1-\gamma_L\cdot\left(\tilde{R}_{t+j+1}^K-1\right)\right]\right]}. \tag{A.7}$$

We can simplify this expression as

$$\frac{P_t^*}{P_t} = \left(\frac{F_t}{H_t}\right)^{\frac{1}{1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)}}, \tag{A.8}$$

where F_t and H_t are recursively written as

$$\begin{aligned} F_t &= (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right)\left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}\left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}}\left(\frac{P_t^K}{P_t}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}F_{t+1}\right], \\ H_t &= \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-1}\frac{Y_t}{A_t\bar{N}_t}\left[1-\gamma_L\cdot\left(\tilde{R}_{t+1}^K-1\right)\right] + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon-1}H_{t+1}\right]. \end{aligned} \tag{A.9}$$

Using (A.9), we obtain the following equilibrium price-resetting condition:

$$\frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]}. \tag{A.10}$$

We now rewrite equation (10) as

$$1 = \beta \cdot \mathbb{E}_t \left[\frac{R^S_{t+1}}{\Pi_{t+1} G A_{t+1} G N} \cdot \frac{\left(\frac{C_t}{A_t \bar{N}_t} \right)}{\left(\frac{C_{t+1}}{A_{t+1} \bar{N}_{t+1}} \right)} \right].$$

Since R^S_{t+1} depends on bonds return R^{HB}_{t+1} and loans return R^K_{t+1} while shares of savings that flow into bonds $(1 - \lambda^K_t)$ and loans (λ^K_t) are endogenous, we start from analyzing R^{HB}_{t+1}. We can rewrite the aggregate return indices as functions of the bond yields $\{YD^f_t\}^F_{f=1}$ as

$$R^j_t = \sum_{f=0}^{F-1} \lambda^{j,f+1}_{t-1} \frac{\left(YD^f_t\right)^{-f}}{\left(YD^{f+1}_{t-1}\right)^{-(f+1)}}, \quad j \in \{H, G, CB\},$$

and also the household's bond portfolio share as

$$\lambda^{HB,f}_t = \left(\frac{\mathbb{E}_t \left[\frac{\beta \cdot z^f_t}{\Pi_{t+1} \cdot G A_{t+1} \cdot G N} \cdot \frac{\left(\frac{C_t}{A_t \bar{N}_t} \right)}{\left(\frac{C_{t+1}}{A_{t+1} \bar{N}_{t+1}} \right)} \cdot \frac{\left(YD^{f-1}_{t+1}\right)^{-(f-1)}}{\left(YD^f_t\right)^{-f}} \right]}{\Phi^B_t} \right)^{\kappa_B}, \quad \forall f,$$

$$\Phi^B_t = \left[\sum_{j=1}^{F} \mathbb{E}_t \left[\frac{\beta \cdot z^j_t}{\Pi_{t+1} \cdot G A_{t+1} \cdot G N} \cdot \frac{\left(\frac{C_t}{A_t \bar{N}_t} \right)}{\left(\frac{C_{t+1}}{A_{t+1} \bar{N}_{t+1}} \right)} \cdot \frac{\left(YD^{j-1}_{t+1}\right)^{-(j-1)}}{\left(YD^j_t\right)^{-j}} \right]^{\kappa_B} \right]^{\frac{1}{\kappa_B}}.$$

Now we find the equilibrium condition for the bond shares of the agents. Using the bond market equilibrium condition (i.e., (17)), we obtain

$$\lambda^{HB,f}_t = \frac{B^{G,f}_t + B^{CB,f}_t}{B^G_t + B^{CB}_t} = \frac{\lambda^{G,f}_t B^G_t + \lambda^{CB,f}_t B^{CB}_t}{B^G_t + B^{CB}_t}. \tag{A.11}$$

We can rearrange the previous expression as

$$\lambda_t^{CB,f} = \lambda_t^{HB,f} + \left(\lambda_t^{HB,f} - \lambda_t^{G,f}\right) \cdot \frac{B_t^G}{B_t^{CB}}. \tag{A.12}$$

Summing from $f = 2$ to F, and using $\sum_{f=2}^{F} \lambda_t^{j,f} = 1 - \lambda_t^{j,1},\ j \in \{H, G, CB\}$ we obtain

$$\sum_{f=2}^{F} \lambda_t^{CB,f} = 1 - \lambda_t^{HB,1} + \left(\lambda_t^{G,1} - \lambda_t^{HB,1}\right) \cdot \frac{B_t^G}{B_t^{CB}}. \tag{A.13}$$

Plugging (A.13) into (A.12) and after some rearrangements, we obtain

$$\lambda_t^{CB,f} = \frac{\lambda_t^{HB,f}\left(\lambda_t^{CB,1} - \lambda_t^{G,1}\right) - \lambda_t^{G,f}\left(\lambda_t^{CB,1} - \lambda_t^{HB,1}\right)}{\lambda_t^{HB,1} - \lambda_t^{G,1}}, \quad f > 1. \tag{A.14}$$

Now, we can obtain an expression for the central bank's bond holdings using (A.13) as

$$B_t^{CB} = \left(\frac{\lambda_t^{HB,1} - \lambda_t^{G,1}}{\lambda_t^{CB,1} - \lambda_t^{HB,1}}\right) \cdot B_t^G. \tag{A.15}$$

Combining (17) and (A.15), we obtain

$$\frac{B_t^H}{A_t \bar{N}_t} = -\left(\frac{\lambda_t^{CB,1} - \lambda_t^{G,1}}{\lambda_t^{CB,1} - \lambda_t^{HB,1}}\right) \cdot \frac{B_t^G}{A_t \bar{N}_t}. \tag{A.16}$$

Combining $L_t = \lambda_t^K S_t$ and $B_t^H = (1 - \lambda_t^K) S_t$ with $L_t = \gamma_L \cdot \left(1 + \zeta^F\right) \cdot P_t Y_t$, we obtain

$$\frac{B_t^H}{A_t \bar{N}_t P_t} = \gamma_L \cdot \left(1 + \zeta^F\right) \cdot \left(\frac{1 - \lambda_t^K}{\lambda_t^K}\right) \left(\frac{Y_t}{A_t \bar{N}_t}\right). \tag{A.17}$$

Combining equation (A.16) and equation (A.17), we get the following equation

$$-\left(\frac{\lambda_t^{CB,1} - \lambda_t^{G,1}}{\lambda_t^{CB,1} - \lambda_t^{HB,1}}\right) \cdot \frac{B_t^G}{A_t \bar{N}_t P_t} = \gamma_L \cdot \left(1 + \zeta^F\right) \cdot \left(\frac{1 - \lambda_t^K}{\lambda_t^K}\right) \left(\frac{Y_t}{A_t \bar{N}_t}\right). \tag{A.18}$$

A.1.2 Conventional Policy in Section 2.9.1

Using the bond market equilibrium (i.e., (17)) with $\sum_{f=2}^{F} \lambda_t^{HB,f} = 1 - \lambda_t^{HB,1}$, we get

$$B_t^H = -\frac{\sum_{i=2}^{F} \left(B_t^{G,i} + B_t^{CB,i} \right)}{1 - \lambda_t^{HB,1}}. \tag{A.19}$$

Combining (A.19) with (4), (17), and (23c), we obtain the equilibrium set of equations,

$$\frac{\lambda_t^{HB,f}}{1 - \lambda_t^{HB,1}} = \frac{\dfrac{B_t^{G,f}}{A_t \bar{N}_t P_t} + \dfrac{\overline{B^{CB,f}}}{A\bar{N}P}}{\sum_{i=2}^{F} \left(\dfrac{B_t^{G,i}}{A_t \bar{N}_t P_t} + \dfrac{\overline{B^{CB,i}}}{A\bar{N}P} \right)}, \quad \forall f > 1. \tag{A.20}$$

Combining (A.17), (A.19), and (23c) yields the following equilibrium equation:

$$-\frac{\sum_{i=2}^{F} \left(\dfrac{B_t^{G,i}}{A_t \bar{N}_t P_t} + \dfrac{\overline{B^{CB,i}}}{A\bar{N}P} \right)}{1 - \lambda_t^{HB,1}} = \gamma_L \cdot \left(1 + \zeta^F\right) \cdot \left(\frac{1 - \lambda_t^K}{\lambda_t^K} \right) \left(\frac{Y_t}{A_t \bar{N}_t} \right), \tag{A.21}$$

where normalized bond positions of the central bank are exogenously given. Finally, combining (A.20) and (A.21), we finally obtain

$$-\left(\frac{B_t^{G,f}}{A_t \bar{N}_t P_t} + \frac{\overline{B^{CB,f}}}{A\bar{N}P} \right) \cdot \left(\lambda_t^{HB,f} \right)^{-1} = \gamma_L \cdot \left(1 + \zeta^F\right) \cdot \left(\frac{1 - \lambda_t^K}{\lambda_t^K} \right) \left(\frac{Y_t}{A_t \bar{N}_t} \right), \quad \forall f > 1. \tag{A.22}$$

A.1.3 Steady-State Derivations in Section 3.1

At the steady state, the central bank decides the level of bond holdings of maturities $B^{CB,f}$ that it wants to hold. It can be calibrated to match the data of the central bank's balance sheet. Given $\left\{\lambda^{CB,f}\right\}$ and the size of its portfolio B^{CB}, which is ζ^{CB} fraction of total government bond issuance satisfying $B^{CB} = \zeta^{CB} \cdot B^G$, we obtain the steady state households' bond shares as

$$\lambda^{HB,f} = \frac{\lambda^{G,f} + \lambda^{CB,f} \cdot \zeta^{CB}}{1 + \zeta^{CB}}. \tag{A.23}$$

From the definition of R^{HB} we have

$$\sum_{f=1}^{F} \lambda^{HB,f} \cdot \left(\frac{R^f}{R^{HB}}\right) = 1,$$

which together with equation (3) can be rearranged as:

$$\lambda^{HB,f} = \left(\frac{z^f \cdot \dfrac{R^f}{R^{HB}}}{\tilde{\Phi}^B}\right)^{\kappa_B}, \ \forall f, \ \text{ with } \ \tilde{\Phi}^B = \left[\sum_{j=1}^{F} \left[z^j \cdot \frac{R^j}{R^{HB}}\right]^{\kappa_B}\right]^{\frac{1}{\kappa_B}}. \tag{A.24}$$

The above (A.23) and (A.24) jointly determine the steady state yields and household shares. Unfortunately, there is no analytical expression for them and we have to solve for the steady state values numerically. How we proceed, relying on simple iterations:

1. Assume some initial guess for $\left\{\frac{R^{f,guess}}{R^{HB}}\right\}_{f=1}^{F}$.
2. Construct $\tilde{\Phi}^{B,old}$ using previous guess with $\tilde{\Phi}^B$ in equation (A.24).
3. Update estimates on $\left\{\frac{R^f}{R^{HB}}\right\}_{f=1}^{F}$ with the following rules:

$$\frac{R^{1,new}}{R^{HB}} = \frac{1 - \sum_{f=2}^{F} \lambda^{HB,f}\left(\dfrac{R^f}{R^{HB}}\right)}{\lambda^{HB,1}}, \quad \frac{R^{f,new}}{R^{HB}} = \left(\lambda^{HB,f}\right)^{\frac{1}{\kappa_B}} \left(z^f\right)^{-1} \tilde{\Phi}^{B,old}, \quad f > 1.$$

4. Construct new shares of households $\lambda^{HB,f,new}$ by plugging $\left\{\frac{R^{f,new}}{R^{HB}}\right\}_{f=1}^{F}$ into (A.24). Compute the discrepancy between these shares and the true ones found in (A.23). If the error is big, set $\frac{R^{f,guess}}{R^{HB}} = \frac{R^{f,new}}{R^{HB}}$ and repeat from Step 2 until convergence.

Using (7) and (10), we obtain

$$R^{HB} = \frac{\beta^{-1}\Pi \cdot GA \cdot GN}{1-\lambda^K} - \frac{\lambda^K}{1-\lambda^K} R^K. \tag{A.25}$$

We can rewrite R^G as

$$R^G = \Xi \cdot R^{HB}, \ \ \Xi = \sum_{f=1}^{F} \lambda^{G,f} \cdot \left(\frac{R^f}{R^{HB}}\right), \tag{A.26}$$

and from (A.25), it becomes

$$R^G = \Xi \cdot \left[\frac{\beta^{-1}\Pi \cdot GA \cdot GN}{1-\lambda^K} - \frac{\lambda^K}{1-\lambda^K} R^K \right]. \tag{A.27}$$

Using (6), we obtain an expression for the steady-state share of loans as

$$\lambda^K = \frac{\left(z^K \cdot \frac{R^K}{R^{HB}}\right)^{\kappa_S}}{1+\left(z^K \cdot \frac{R^K}{R^{HB}}\right)^{\kappa_S}}. \tag{A.28}$$

Further combining (A.25) and (A.28), we obtain

$$\frac{\lambda^K}{1-\lambda^K} \equiv \left(z^K \cdot \frac{R^K}{R^{HB}}\right)^{\kappa_S} = \frac{\beta^{-1} \cdot \Pi \cdot GA \cdot GN - R^{HB}}{R^K - \beta^{-1} \cdot \Pi \cdot GA \cdot GN} \tag{A.29}$$

The equilibrium government bonds are obtained from its budget constraint (i.e., (20)) and written as

$$\frac{B^G}{A\bar{N}P} = -\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1} \left[\zeta^G + \zeta^F - \zeta^T\right] \left(\frac{Y}{A\bar{N}}\right). \tag{A.30}$$

The model needs the government to be a borrower, so $B^G < 0$ at the steady-state. Also, we would like to match the data in which the government runs primary deficit $\zeta^G + \zeta^F - \zeta^T > 0$. The only way to achieve that is by having $R^G < \Pi \cdot GA \cdot GN$. Combining $B^{CB} = \zeta^{CB} \cdot B^G$, $B^H_t = -\left(B^G_t + B^{CB}_t\right)$, (A.17), (A.26), and (A.29) yields

$$\gamma_L = \left(\frac{1+\zeta^{CB}}{1+\zeta^F}\right) \cdot \left[\zeta^G + \zeta^F - \zeta^T\right] \cdot \left(1 - \frac{\Xi}{\Pi \cdot GA \cdot GN} \cdot R^{HB}\right)^{-1} \cdot \left[\frac{\beta^{-1} \cdot \Pi \cdot GA \cdot GN - R^{HB}}{R^K - \beta^{-1} \cdot \Pi \cdot GA \cdot GN}\right]. \tag{A.31}$$

(A.29) and (A.31) form a nonlinear system of equations on the unknown steady-states of R^K and R^{HB}. After then, we can then simply back out bond returns as

$$R^f = R^{HB} \cdot \left(\frac{R^f}{R^{HB}}\right).$$

Now that we have found the bond returns, we can recursively obtain the bond yields using

$$YD^f = \left[R^f \cdot \left(YD^{f-1}\right)^{f-1}\right]^{\frac{1}{f}},$$

where $YD^0 = 1$. The price dispersion is given by

$$\Delta = \left[\frac{1-\theta}{1-\theta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}}\right]\left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)}. \tag{A.32}$$

From the capital producer's optimization (i.e., (11)), we obtain an expression for P^K

$$\frac{P^K}{P} = \beta^{-1}\cdot GA\cdot GN - (1-\delta). \tag{A.33}$$

The steady state representation of firms' pricing (i.e., (A.9) can be written as

$$F = \xi^F\cdot\left(\frac{C}{A\bar{N}}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}\left(\frac{Y}{A\bar{N}}\right)^{\frac{\eta+1}{\eta+\alpha}}, \tag{A.34}$$

$$H = \xi^H\cdot\left(\frac{C}{A\bar{N}}\right)^{-1}\left(\frac{Y}{A\bar{N}}\right), \tag{A.35}$$

$$\text{with}\quad \xi^F = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\left[1-\theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]^{-1}\left(\frac{(1+\zeta^F)^{-1}\epsilon}{\epsilon-1}\right)\left(\frac{P^K}{P}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)},$$

$$\xi^H = \left[1-\gamma_L\cdot\left(\frac{R^K}{R^S}-1\right)\right]\cdot\left[1-\theta\beta\cdot\Pi^{\epsilon-1}\right]^{-1}.$$

Using (A.34), (A.35) and (A.10), we obtain

$$\left(\frac{C}{A\bar{N}}\right)^{\left(\frac{(1-\alpha)\eta}{\eta+\alpha}\right)} = \xi^Y\cdot\left(\frac{Y}{A\bar{N}}\right)^{-\left(\frac{1-\alpha}{\eta+\alpha}\right)},\quad\text{with}\quad \xi^Y = \left(\frac{\xi^H}{\xi^F}\right)\left(\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]}. \tag{A.36}$$

Combining (A.36) and (A.4), we obtain

$$\frac{K}{A\bar{N}} = \xi^K\cdot\left(\frac{Y}{A\bar{N}}\right),\quad\text{with}\quad \xi^K = \alpha\cdot(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\cdot GA\cdot GN\cdot\left(\frac{P^K}{P}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)}\Delta\cdot\xi^Y. \tag{A.37}$$

Plugging (A.37) into the aggregate resource constraint, we obtain

$$\frac{C}{A\bar{N}} = \xi^C\cdot\left(\frac{Y}{A\bar{N}}\right),\quad\text{with}\quad \xi^C = (1-\zeta^G) - \xi^K\cdot\left[1-\left(\frac{1-\delta}{GA\cdot GN}\right)\right]. \tag{A.38}$$

Combining (A.36) and (A.38), we obtain

$$\frac{Y}{A\bar{N}} = \left(\xi^{Y}\right)^{\left(\frac{1}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta+1}\right)} \left(\xi^{C}\right)^{-\left(\frac{\eta}{\eta+1}\right)}. \tag{A.39}$$

A.1.4 Log-linearization

We start by log-linearizing the equations that are common to the conventional policy model and the yield-curve-control one, then derive the ones that are different. Log-linearize (33),

$$\hat{ga}_t = \hat{\varepsilon}_t^A, \;\; \hat{\zeta}_t^G = \frac{a^G}{1+a^G} \cdot \hat{u}_t^G, \;\; \hat{\zeta}_t^T = \frac{a^T}{1+a^T} \cdot \hat{u}_t^T. \tag{A.40}$$

Equations (38) and (10) with the help of (A.40) can be linearized as

$$\hat{c}_t = \left[\left(1-\zeta^G\right) \cdot \frac{Y}{C}\right]\left[\hat{y}_t - \frac{1}{1+a^G} \cdot \hat{u}_t^G\right] + \left[\frac{1-\delta}{GA \cdot GN}\frac{K}{C}\right](\hat{k}_t - \hat{\varepsilon}_t^A) - \frac{K}{C}\hat{k}_{t+1}, \tag{A.41}$$

$$\hat{c}_t = \mathbb{E}_t\left[\hat{c}_{t+1} - \left(\hat{r}_{t+1}^S - \hat{\pi}_{t+1}\right)\right], \tag{A.42}$$

where we use (A.40) to solve for $\hat{\zeta}_t^G$ and $\hat{ga}_t$. Plugging (A.41) into (A.42), we obtain the following dynamic IS equation for output $\hat{y}_t$.

$$\begin{aligned}\hat{y}_t = \mathbb{E}_t\Bigg[&\hat{y}_{t+1} - \left[\frac{(1-\zeta^G)^{-1}(1-\delta)}{GA \cdot GN} \cdot \frac{K}{Y}\right](\hat{k}_t - \hat{\varepsilon}_t^A) + (1-\zeta^G)^{-1}\left[1 + \frac{1-\delta}{GA \cdot GN}\right]\frac{K}{Y}\hat{k}_{t+1} \\ &- (1-\zeta^G)^{-1}\frac{K}{Y}\hat{k}_{t+2} - (1-\zeta^G)^{-1}\frac{C}{Y}\left(\hat{r}_{t+1}^S - \hat{\pi}_{t+1}\right) + \frac{1-\rho_G}{1+a^G} \cdot \hat{u}_t^G\Bigg].\end{aligned} \tag{A.43}$$

Linearizing the household's bond portfolio conditions (i.e., (3)) yields

$$\hat{\lambda}_t^{HB,f} = \kappa^B \mathbb{E}_t\left[\hat{z}_t^f - \hat{\pi}_{t+1} - \hat{ga}_{t+1} + \hat{c}_t - \hat{c}_{t+1} - (f-1)\hat{yd}_{t+1}^{f-1} + f\hat{yd}_t^f - \hat{\phi}_t^B\right], \tag{A.44}$$

where

$$\begin{aligned}\hat{\phi}_t^B = {}& \mathbb{E}_t\left(-\hat{\pi}_{t+1} - \hat{ga}_{t+1} + \hat{c}_t - \hat{c}_{t+1}\right) + \sum_{j=1}^{F}\left[\frac{\beta z^j\left(YD^{j-1}\right)^{-(j-1)}}{\Pi \cdot GA \cdot GN \cdot \Phi^B\left(YD^j\right)^{-j}}\right]^{\kappa^B}\hat{z}_t^j \\ &+ \sum_{j=1}^{F} j\left[\frac{\beta z^j\left(YD^{j-1}\right)^{-(j-1)}}{\Pi \cdot GA \cdot GN \cdot \Phi^B\left(YD^j\right)^{-j}}\right]^{\kappa^B}\hat{yd}_t^j - \sum_{j=0}^{F-1} j\left[\frac{\beta z^{j+1}\left(YD^j\right)^{-j}}{\Pi \cdot GA \cdot GN \cdot \Phi^B\left(YD^{j+1}\right)^{-(j+1)}}\right]^{\kappa^B}\mathbb{E}_t(\hat{yd}_{t+1}^j).\end{aligned} \tag{A.45}$$

Combining (A.44) and (A.45), we obtain the following expression for $\hat{\lambda}_t^{HB,f}$:

$$\hat{\lambda}_t^{HB,f} = \sum_{j=1}^{F} \Psi_1^{fj} \hat{z}_t^j + \sum_{j=1}^{F} \Psi_2^{fj} \hat{yd}_t^j + \sum_{j=1}^{F} \Psi_3^{fj} \mathbb{E}_t \left[\hat{yd}_{t+1}^j \right], \tag{A.46}$$

where

$$\Psi_1^{fj} = \begin{cases} \left[1 - \left[\dfrac{\beta \cdot z^j \left(YD^{j-1} \right)^{-(j-1)}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^j \right)^{-j}} \right]^{\kappa^B} \right] \cdot \kappa^B & \text{, if } f = j, \\ - \left[\dfrac{\beta \cdot z^j \left(YD^{j-1} \right)^{-(j-1)}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^j \right)^{-j}} \right]^{\kappa^B} \cdot \kappa^B & \text{, if } f \neq j, \end{cases}$$

$$\Psi_2^{fj} = j \cdot \Psi_1^{fj},$$

$$\Psi_3^{fj} = \begin{cases} -j \cdot \left[1 - \left[\dfrac{\beta \cdot z^{j+1} \left(YD^j \right)^{-j}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^{j+1} \right)^{-(j+1)}} \right]^{\kappa^B} \right] \cdot \kappa^B & \text{, if } j = f-1, \\ j \cdot \left[\dfrac{\beta \cdot z^{j+1} \left(YD^j \right)^{-j}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^{j+1} \right)^{-(j+1)}} \right]^{\kappa^B} \cdot \kappa^B & \text{, if } j \neq f-1, \\ 0 & \text{, if } j = F. \end{cases}$$

We can put the system of F equation in matrix format as

$$\overrightarrow{\hat{\lambda}_t^{HB}} = \Psi_1 \cdot \overrightarrow{\hat{z}}_t + \Psi_2 \cdot \overrightarrow{\hat{yd}_t} + \Psi_3 \cdot \mathbb{E}_t \left[\overrightarrow{\hat{yd}_{t+1}} \right], \tag{A.47}$$

where $\{\Psi_1, \Psi_2, \Psi_3\}$ are matrices containing elements of $\left\{\Psi_1^{fj}, \Psi_2^{fj}, \Psi_3^{fj}\right\}$, with f representing rows and j columns. Linearizing equations (34) and (35) yields

$$\overrightarrow{\hat{\lambda}_t^G} = \widetilde{\Xi} \cdot \overrightarrow{\hat{\tilde{u}}_t^B}, \quad \overrightarrow{\hat{\tilde{u}}_t^B} = \mathcal{T}^B \cdot \overrightarrow{\hat{u}_t^B}.$$

where $\widetilde{\Xi}$ is a matrix whose elements $\widetilde{\Xi}_{fj}$ (f-rows, j-columns) are

$$\widetilde{\Xi}_{fj} = \begin{cases} 0 & \text{, if } f = 1 \ \& \ j = f, \\ 1 - \lambda^{G,f} & \text{, if } f \geq 2 \ \& \ j = f, \\ -\lambda^{G,j} & \text{, if } j \neq f, \end{cases}$$

and similarly $\mathcal{T}^B$ is a matrix containing elements τ_{fj}^B from (35). By defining $\Xi = \widetilde{\Xi} \cdot \mathcal{T}^B$, we can combine the previous two equations to obtain

$$\overrightarrow{\hat{\lambda}_t^G} = \Xi \cdot \overrightarrow{\hat{u}_t^B}. \tag{A.48}$$

Therefore, with the help of (A.48), we obtain

$$\overrightarrow{\hat{b}_t^G} = \Xi \cdot \overrightarrow{\hat{u}_t^B} + \overrightarrow{1}_{Fx1} \cdot \hat{b}_t^G, \tag{A.49}$$

where $\overrightarrow{1}_{Fx1}$ is a unit vector of size F. Log-linearizing the household's stochastic discount factor yields:

$$\hat{q}_{t,t+1} = \hat{c}_t - \hat{c}_{t+1} - \hat{\pi}_{t+1} - \hat{ga}_{t+1}. \tag{A.50}$$

Log-linearizing Φ_t^S in the household's portfolio between loans and bonds (i.e., (6)), we obtain

$$\hat{\phi}_t^S = \mathbb{E}_t\left[q_{t,t+1}\right] + \frac{\left(z^B R^{HB}\right)^{\kappa^S}}{\left(z^B R^{HB}\right)^{\kappa^S} + \left(z^K R^K\right)^{\kappa^S}} \mathbb{E}_t\left[\hat{r}_{t+1}^{HB}\right] + \frac{\left(z^K R^K\right)^{\kappa^S}}{\left(z^B R^{HB}\right)^{\kappa^S} + \left(z^K R^K\right)^{\kappa^S}} \left(\hat{z}_t^K + \mathbb{E}_t\left[\hat{r}_{t+1}^K\right]\right). \tag{A.51}$$

Log-linearizing the household's portfolio decision between loans and bonds (i.e., (6)) and making use of the previous expression (i.e., (A.51)), we obtain

$$\begin{aligned}
\hat{\lambda}_t^K &= \kappa^S \cdot \left[\frac{\left(z^B R^{HB}\right)^{\kappa^S}}{\left(z^B R^{HB}\right)^{\kappa^S} + \left(z^K R^K\right)^{\kappa^S}}\right] \left(\hat{z}_t^K + \mathbb{E}_t\left[\hat{r}_{t+1}^K - \hat{r}_{t+1}^{HB}\right]\right) \\
&= \kappa^S \left(1 - \lambda^K\right) \left(\hat{z}_t^K + \mathbb{E}_t\left[\hat{r}_{t+1}^K - \hat{r}_{t+1}^{HB}\right]\right).
\end{aligned} \tag{A.52}$$

By linearizing the formula for the effective savings rate of the household (i.e., (7)), we obtain

$$\hat{r}_t^S = \frac{\lambda^K \left(R^K - R^{HB}\right)}{R^S} \hat{\lambda}_{t-1}^K + \frac{(1-\lambda^K) R^{HB}}{R^S} \hat{r}_t^{HB} + \frac{\lambda^K R^K}{R^S} \hat{r}_t^K. \tag{A.53}$$

Log-linearizing the effective bond rates of households, government, and central bank,

$$\hat{r}_t^j = \sum_{f=1}^{F} \frac{\lambda^{j,f} \left(YD^{f-1}\right)^{-(f-1)}}{R^j \left(YD^f\right)^{-f}} \cdot \left[\hat{\lambda}_{t-1}^{j,f} - (f-1)\hat{yd}_t^{f-1} + f \hat{yd}_{t-1}^f\right], \quad j \in \{HB, G, CB\},$$

with which we can express these equations on matrix format as

$$\hat{r}_t^j = \Psi^{j,4} \cdot \overrightarrow{\hat{\lambda}_{t-1}^j} - \Psi^{j,5} \cdot \overrightarrow{\hat{yd}_t} + \Psi^{j,6} \cdot \overrightarrow{\hat{yd}_{t-1}}\,, \qquad j \in \{HB, G, CB\}\,, \tag{A.54}$$

where $\{\Psi^{j,4}, \Psi^{j,5}, \Psi^{j,6}\}$ are $1xF$-sized matrices whose elements are defined as:

$$\Psi_{1f}^{j,4} = \frac{\lambda^{j,f}\left(YD^{f-1}\right)^{-(f-1)}}{R^j\left(YD^f\right)^{-f}}, \quad \Psi_{1f}^{j,6} = \frac{\lambda^{j,f}\left(YD^{f-1}\right)^{-(f-1)}}{R^j\left(YD^f\right)^{-f}}f.$$

$$\Psi_{1f}^{j,5} = \begin{cases} \dfrac{\lambda^{j,f+1}\left(YD^f\right)^{-f}}{R^j\left(YD^{f+1}\right)^{-(f+1)}}f & \text{, if } f < F, \;\; j \in \{HB, G, CB\}\,, \\ 0 & \text{, if } f = F, \end{cases}$$

By plugging (A.48) into $\hat{r}^G$ in (A.54), we obtain

$$\hat{r}_t^G = \Psi^{G,4} \cdot \Xi \cdot \overrightarrow{\hat{u}_{t-1}^B} - \Psi^{G,5} \cdot \overrightarrow{\hat{yd}_t} + \Psi^{G,6} \cdot \overrightarrow{\hat{yd}_{t-1}}. \tag{A.55}$$

By plugging (A.47) into $\hat{r}^{HB}$ in (A.54), we obtain

$$\hat{r}_t^{HB} = \Psi^{HB,4}\Psi^1 \cdot \overrightarrow{\hat{z}_{t-1}} + \left[\Psi^{HB,4}\Psi^2 + \Psi^{HB,6}\right] \cdot \overrightarrow{\hat{yd}_{t-1}} + \Psi^{HB,4}\Psi^3 \cdot \mathbb{E}_{t-1}\left[\overrightarrow{\hat{yd}_t}\right] - \Psi^{HB,5} \cdot \overrightarrow{\hat{yd}_t}. \tag{A.56}$$

Taking the expectation operator $\mathbb{E}_t$ on the previous equation (A.56), we obtain

$$\mathbb{E}_t\left[\hat{r}_{t+1}^{HB}\right] = \Psi^{HB,4}\Psi^1 \cdot \overrightarrow{\hat{z}_t} + \left[\Psi^{HB,4}\Psi^2 + \Psi^{HB,6}\right] \cdot \overrightarrow{\hat{yd}_t} + \left[\Psi^{HB,4}\Psi^3 - \Psi^{HB,5}\right]\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right]. \tag{A.57}$$

By plugging (A.52) and (A.57) into (A.53), we obtain the expected effective savings rate as follows.

$$\mathbb{E}_t\left[\hat{r}_{t+1}^S\right] = \Psi^7 \overrightarrow{\hat{z}}_t + \Psi^8 \overrightarrow{\hat{yd}_t} + \Psi^9 \mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{10}\hat{r}_{t+1}^K + \Psi^{11}\hat{z}_t^K, \tag{A.58}$$

where

$$\Psi^{7} = \Psi^{HB,4}\Psi^{1}\left[\frac{(1+\kappa^{S}\lambda^{K})(1-\lambda^{K})R^{HB} - \kappa^{S}(1-\lambda^{K})\lambda^{K}R^{K}}{R^{S}}\right],$$

$$\Psi^{8} = \left[\Psi^{HB,4}\Psi^{2} + \Psi^{HB,6}\right]\left[\frac{(1+\kappa^{S}\lambda^{K})(1-\lambda^{K})R^{HB} - \kappa^{S}(1-\lambda^{K})\lambda^{K}R^{K}}{R^{S}}\right],$$

$$\Psi^{9} = \left[\Psi^{HB,4}\Psi^{3} - \Psi^{HB,5}\right]\left[\frac{(1+\kappa^{S}\lambda^{K})(1-\lambda^{K})R^{HB} - \kappa^{S}(1-\lambda^{K})\lambda^{K}R^{K}}{R^{S}}\right],$$

$$\Psi^{10} = \frac{\left[1+\kappa^{S}(1-\lambda^{K})\right]\lambda^{K}R^{K} - \kappa^{S}(1-\lambda^{K})\lambda^{K}R^{HB}}{R^{S}}, \quad \Psi^{11} = \frac{\kappa^{S}\lambda^{K}(1-\lambda^{K})\left(R^{K}-R^{HB}\right)}{R^{S}}$$

Plugging back the expression of the household's expected bonds rate (i.e., (A.57)) into her portfolio decision between loans and bonds (i.e., (A.52)), we obtain

$$\hat{\lambda}_t^{K} = \kappa^{S}\left(1-\lambda^{K}\right)\left(\hat{z}_t^{K} + \hat{r}_{t+1}^{K}\right) - \Psi^{12}\cdot\overrightarrow{\hat{z}_t} - \Psi^{13}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{14}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right], \tag{A.59}$$

where $\Psi^{12} = \kappa^{S}(1-\lambda^{K})\Psi^{HB,4}\Psi^{1}$, $\Psi^{13} = \kappa^{S}(1-\lambda^{K})\left[\Psi^{HB,4}\Psi^{2} + \Psi^{HB,6}\right]$, and $\Psi^{13} = \kappa^{S}(1-\lambda^{K})\left[\Psi^{HB,4}\Psi^{2} + \Psi^{HB,6}\right]$. If we linearize the loan aggregation (A.4) and use (A.40), we obtain

$$\hat{k}_t = \hat{\varepsilon}_t^{A} + \left(\frac{\eta+1}{\eta+\alpha}\right)\hat{y}_t - \left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)\cdot\left[\hat{p}_t^{K} - \hat{c}_t\right]. \tag{A.60}$$

Combining (A.41) and the above (A.60), we obtain

$$\begin{aligned} p_t^{K} = &\left[(1-\zeta^{G})\frac{Y}{C} + \frac{\eta+1}{\eta(1-\alpha)}\right]\cdot\hat{y}_t - \left[(1-\zeta^{G})\cdot\frac{Y}{C}\right]\left(\frac{1}{1+a^{G}}\right)\cdot u_t^{G} \\ &+ \left[\frac{1-\delta}{GA\cdot GN}\cdot\frac{K}{C} - \frac{\eta+\alpha}{\eta(1-\alpha)}\right]\cdot\left[\hat{k}_t - \hat{\varepsilon}_t^{A}\right] - \frac{K}{C}\cdot\hat{k}_{t+1}. \end{aligned} \tag{A.61}$$

If we linearize the supply block (i.e., (A.9), and (A.10)), we obtain

$$\begin{aligned} \hat{f}_t = &\left[1-\theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]\left(\frac{\eta+1}{\eta+\alpha}\right)\left[\hat{y}_t + \alpha\mathbb{E}_t\left[\hat{q}_{t,t+1} + \hat{p}_t^{K} - \hat{c}_t\right]\right] \\ &+ \theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\mathbb{E}_t\left[\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)\hat{\pi}_{t+1} + \hat{f}_{t+1}\right], \\ \hat{h}_t = &\left[1-\theta\beta\Pi^{\epsilon-1}\right]\left[\hat{y}_t - \hat{c}_t - \left(\frac{\gamma_L\cdot R^{K}}{R^{S} - \gamma_L\cdot(R^{K}-R^{S})}\right)\cdot\left[\hat{r}_{t+1}^{K} + \mathbb{E}_t\left[\hat{q}_{t,t+1}\right]\right]\right] \\ &+ \theta\beta\Pi^{\epsilon-1}\mathbb{E}_t\left[(\epsilon-1)\hat{\pi}_{t+1} + \hat{h}_{t+1}\right], \end{aligned} \tag{A.62}$$

$$\hat{f}_t - \hat{h}_t = \left[1 + \epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\left(\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)\hat{\pi}_t. \tag{A.63}$$

Combining (A.42), (A.50), and (A.60) with (A.62), we obtain:

$$\begin{aligned}\hat{f}_t = &- \Psi^{16}\cdot\overrightarrow{\hat{z}_t} - \Psi^{17}\cdot\hat{z}_t^K - \Psi^{18}\cdot\left[\hat{k}_t - \hat{\varepsilon}_t^A\right] + \Psi^{19}\cdot\hat{y}_t - \Psi^{20}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{21}\cdot\hat{r}_{t+1}^K \qquad (A.64)\\ &- \Psi^{22}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{23}\cdot\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{24}\cdot\mathbb{E}_t\left[\hat{f}_{t+1}\right].\\ \hat{h}_t = &\Psi^{26}\cdot\overrightarrow{\hat{z}_t} + \Psi^{27}\cdot\hat{z}_t^K - \Psi^{28}\cdot\left[\hat{k}_t - \hat{\varepsilon}_t^A\right] + \Psi^{29}\cdot\hat{u}_t^G + \Psi^{30}\cdot\hat{y}_t + \Psi^{31}\cdot\overrightarrow{\hat{yd}_t} \qquad (A.65)\\ &- \Psi^{32}\cdot\hat{r}_{t+1}^K + \Psi^{33}\cdot\hat{k}_{t+1} + \Psi^{34}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{35}\cdot\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{36}\cdot\mathbb{E}_t\left[\hat{h}_{t+1}\right].\end{aligned}$$

where

$$\Psi^{15} = \left[1 - \theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]\left(\frac{\eta+1}{\eta+\alpha}\right),$$

$$\Psi^{16} = \alpha\Psi^{15}\Psi^{7},$$

$$\Psi^{17} = \alpha\Psi^{15}\Psi^{11},$$

$$\Psi^{18} = \Psi^{15}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta}\right),$$

$$\Psi^{19} = \Psi^{15}\left[1 + \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta+1}{\eta}\right)\right],$$

$$\Psi^{20} = \alpha\Psi^{15}\Psi^{8},$$

$$\Psi^{21} = \alpha\Psi^{15}\Psi^{10},$$

$$\Psi^{22} = \alpha\Psi^{15}\Psi^{9},$$

$$\Psi^{23} = \theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right),$$

$$\Psi^{24} = \theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)},$$

$$\Psi^{25} = \left[1 - \theta\beta\Pi^{\epsilon-1}\right]\left(\frac{\gamma_L\cdot R^K}{R^S - \gamma_L\cdot(R^K - R^S)}\right),$$

$$\Psi^{26} = \Psi^{25}\Psi^{7},$$

$$\Psi^{27} = \Psi^{25}\Psi^{11},$$

$$\Psi^{28} = \left[1 - \theta\beta\Pi^{\epsilon-1}\right]\left[\frac{1-\delta}{GA\cdot GN}\cdot\frac{K}{C}\right],$$

$$\Psi^{29} = \left[1 - \theta\beta\Pi^{\epsilon-1}\right](1-\zeta^G)\frac{Y}{C}\left(\frac{1}{1+a^G}\right),$$

$$\Psi^{30} = \left[1 - \theta\beta\Pi^{\epsilon-1}\right]\left[1 - (1-\zeta^G)\frac{Y}{C}\right],$$

$$\Psi^{31} = \Psi^{25}\Psi^{8},$$

$$\Psi^{32} = \Psi^{25}\cdot\left(1 - \Psi^{10}\right),$$

$$\Psi^{33} = \left[1 - \theta\beta\Pi^{\epsilon-1}\right]\frac{K}{C},$$

$$\Psi^{34} = \Psi^{25}\Psi^{9},$$

$$\Psi^{35} = \theta\beta\Pi^{\epsilon-1}(\epsilon-1),$$

$$\Psi^{36} = \theta\beta\Pi^{\epsilon-1}.$$

Linearizing the government's budget constraint (i.e., (20)) yields

$$\begin{aligned}\hat{b}_t^G = &\frac{R^G}{\Pi\cdot GA\cdot GN}\left[\hat{r}_t^G - \hat{\pi}_t - \hat{ga}_t + \hat{b}_{t-1}^G\right] \qquad (A.66)\\ &- \left[\zeta^G + \zeta^F - \zeta^T\right]\left(\frac{Y}{B/P}\right)\left[\hat{y}_t + \left(\frac{\zeta^G}{\zeta^G + \zeta^F - \zeta^T}\right)\left(\frac{a^G}{1+a^G}\right)\hat{u}_t^G - \left(\frac{\zeta^T}{\zeta^G + \zeta^F - \zeta^T}\right)\left(\frac{a^T}{1+a^T}\right)\hat{u}_t^T\right].\end{aligned}$$

Using the steady state equilibrium condition (i.e., (A.30)) with (A.40) and (A.55), we can express the previous (A.66) as

$$\hat{b}_t^G = \frac{R^G}{\Pi \cdot GA \cdot GN} \left[\Psi^{G,4} \cdot \Xi \cdot \overrightarrow{\hat{u}_{t-1}^B} - \Psi^{G,5} \cdot \overrightarrow{\hat{yd}_t} + \Psi^{G,6} \cdot \overrightarrow{\hat{yd}_{t-1}} - \hat{\pi}_t - \hat{\varepsilon}_t^A + \hat{b}_{t-1}^G \right] + \left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right) \left[\hat{y}_t + \left(\frac{\zeta^G}{\zeta^G + \zeta^F - \zeta^T}\right)\left(\frac{a^G}{1+a^G}\right)\hat{u}_t^G - \left(\frac{\zeta^T}{\zeta^G + \zeta^F - \zeta^T}\right)\left(\frac{a^T}{1+a^T}\right)\hat{u}_t^T \right]. \tag{A.67}$$

Linearizing the capital producer's optimization condition (i.e., (11)) yields

$$0 = \mathbb{E}_t \left[\hat{q}_{t,t+1} + \hat{\pi}_{t+1} + \left(\frac{P^K/P}{1 - \delta + P^K/P}\right) \hat{p}_{t+1}^K \right]. \tag{A.68}$$

By plugging (A.42) and (A.50) into the previous (A.68) and rearranging, we get

$$\mathbb{E}_t \left[\hat{r}_{t+1}^S - \hat{\pi}_{t+1} \right] = \left(\frac{P^K/P}{1 - \delta + P^K/P}\right) \mathbb{E}_t \left[\hat{p}_{t+1}^K \right]. \tag{A.69}$$

Plugging expressions on the effective savings rate (i.e., (A.58)) and the rental price of capital (i.e., (A.61)) into (A.69) we obtain

$$\hat{r}_{t+1}^K = -\Psi^{37} \cdot \overrightarrow{\hat{z}_t} - \Psi^{38} \cdot \hat{z}_t^K - \Psi^{39} \cdot \overrightarrow{\hat{yd}_t} - \Psi^{40} \cdot \mathbb{E}_t \left[\overrightarrow{\hat{yd}_{t+1}} \right] + \Psi^{41} \cdot \mathbb{E}_t \left[\hat{\pi}_{t+1} \right] + \Psi^{42} \cdot \mathbb{E}_t \left[\hat{y}_{t+1} \right] + \Psi^{43} \cdot \hat{k}_{t+1} - \Psi^{44} \cdot \mathbb{E}_t \left[\hat{k}_{t+2} \right] - \Psi^{45} \cdot \hat{u}_t^G, \tag{A.70}$$

where we defined

$$\Psi^{37} = \left(\Psi^{10}\right)^{-1} \Psi^7, \quad \Psi^{38} = \left(\Psi^{10}\right)^{-1} \Psi^{11}, \quad \Psi^{39} = \left(\Psi^{10}\right)^{-1} \Psi^8, \quad \Psi^{40} = \left(\Psi^{10}\right)^{-1} \Psi^9, \quad \Psi^{41} = \left(\Psi^{10}\right)^{-1},$$

$$\Psi^{42} = \left(\Psi^{10}\right)^{-1} \left(\frac{\frac{P^K}{P}}{1 - \delta + \frac{P^K}{P}}\right) \left[(1 - \zeta^G)\frac{Y}{C} + \left(\frac{\eta + 1}{\eta(1-\alpha)}\right) \right],$$

$$\Psi^{43} = \left(\Psi^{10}\right)^{-1} \left(\frac{\frac{P^K}{P}}{1 - \delta + \frac{P^K}{P}}\right) \left[\frac{1-\delta}{GA \cdot GN}\frac{K}{C} - \left(\frac{\eta + \alpha}{\eta(1-\alpha)}\right) \right],$$

$$\Psi^{44} = \left(\Psi^{10}\right)^{-1} \left(\frac{\frac{P^K}{P}}{1 - \delta + \frac{P^K}{P}}\right) \frac{K}{C},$$

$$\Psi^{45} = \left(\Psi^{10}\right)^{-1} \left(\frac{\frac{P^K}{P}}{1 - \delta + \frac{P^K}{P}}\right) (1 - \zeta^G)\frac{Y}{C} \left(\frac{\rho^G}{1 + a^G}\right).$$

Finally, plugging the effective savings rate (i.e., (A.58)) into the Euler equation (i.e., (A.43)), we obtain

$$\hat{y}_t = \mathbb{E}_t\Big[\hat{y}_{t+1} + \Psi^{46}\cdot\hat{\pi}_{t+1} - \Psi^{47}\cdot\overrightarrow{\hat{z}}_t - \Psi^{48}\cdot\hat{z}_t^K - \Psi^{49}\cdot\overrightarrow{\hat{yd}}_t - \Psi^{50}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] \tag{A.71}$$
$$- \Psi^{51}\cdot\hat{r}_{t+1}^K - \Psi^{52}\cdot(\hat{k}_t - \hat{\varepsilon}_t^A) + \Psi^{53}\cdot\hat{k}_{t+1} - \Psi^{54}\cdot\hat{k}_{t+2} + \Psi^{55}\cdot\hat{u}_t^G\Big],$$

where we defined

$$\begin{aligned}
\Psi^{46} &= (1-\zeta^G)^{-1}\frac{C}{Y}, & \Psi^{52} &= \frac{(1-\zeta^G)^{-1}(1-\delta)}{GA\cdot GN}\frac{K}{Y},\\
\Psi^{47} &= \Psi^{27}\Psi^{7}, & \Psi^{53} &= (1-\zeta^G)^{-1}\left[1+\frac{1-\delta}{GA\cdot GN}\right]\frac{K}{Y},\\
\Psi^{48} &= \Psi^{27}\Psi^{11}, & & \\
\Psi^{49} &= \Psi^{27}\Psi^{8}, & \Psi^{54} &= (1-\zeta^G)^{-1}\frac{K}{Y},\\
\Psi^{50} &= \Psi^{27}\Psi^{9}, & \Psi^{55} &= \frac{1-\rho_G}{1+a^G}.\\
\Psi^{51} &= \Psi^{27}\Psi^{10}, & &
\end{aligned}$$

Linearizing the labor aggregation condition (i.e., (A.5)) yields

$$\hat{n}_t = -\alpha\left(\frac{\eta}{\eta+\alpha}\right)\hat{c}_t + \left(\frac{\eta}{\eta+\alpha}\right)\hat{y}_t + \alpha\left(\frac{\eta}{\eta+\alpha}\right)\cdot\hat{p}_t^K. \tag{A.72}$$

Plugging equation (A.41) and equation (A.61) into equation (A.72), we obtain

$$\hat{n}_t = \left(\frac{\eta}{\eta+\alpha}\right)\left[1+\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta+1}{\eta}\right)\right]\cdot\hat{y}_t - \left(\frac{\alpha}{1-\alpha}\right)\cdot\left[\hat{k}_t - \hat{\varepsilon}_t^A\right]. \tag{A.73}$$

Log-linearization: the conventional policy specific derivations Linearizing the bond market equilibrium condition (i.e., (A.22)), we obtain

$$\hat{\lambda}_t^{HB,f} = \left(\frac{B^{G,f}}{B^{G,f}+B^{CB,f}}\right)\hat{b}_t^{g,f} - \hat{y}_t + \frac{1}{1-\lambda^K}\cdot\hat{\lambda}_t^K,\ \ f\geq 2. \tag{A.74}$$

From $\lambda_t^{HB,1} = 1-\sum_{f=2}^F\lambda_t^{HB,f}$ we obtain

$$\hat{\lambda}_t^{HB,1} = -\sum_{f=2}^F\frac{\lambda^{HB,f}}{\lambda^{HB,1}}\hat{\lambda}_t^{HB,f}. \tag{A.75}$$

We can rearrange the previous expressions (i.e., (A.74) and (A.75)) in the matrix form as

$$\Theta^1 \cdot \overrightarrow{\hat{\lambda}_t^{HB}} = \Theta^2 \cdot \overrightarrow{\hat{b}_t^g} - \Theta^3 \cdot \hat{y}_t + \Theta^4 \cdot \hat{\lambda}_t^K, \tag{A.76}$$

where $\{\Theta^1,\ \Theta^2\}$ are FxF-sized matrices with elements Θ^1_{jf} (row j, column f) and $\{\Theta^3,\ \Theta^4\}$ are $Fx1$ vectors with $j-$element Θ^3_{j1}. We define their elements as

$$\Theta^1_{jf} = \begin{cases} 1 & \text{, if } j = f, \\ \dfrac{\lambda^{HB,f}}{\lambda^{HB,1}} & \text{, if } j = 1\,\&\, f > 1, \end{cases} \qquad \Theta^3_{j1} = \begin{cases} 0 & \text{, if } j = 1, \\ 1 & \text{, otherwise,} \end{cases},$$

$$\Theta^2_{jf} = \begin{cases} \dfrac{B^{G,f}}{B^{G,f} + B^{CB,f}} & \text{, if } j > 1\,\&\, j = f, \\ 0 & \text{, otherwise,} \end{cases} \qquad \Theta^4 = \frac{1}{1-\lambda^K} \cdot \Theta^3 .$$

By inverting Θ^1 in (A.76), we can rewrite (A.76) as

$$\overrightarrow{\hat{\lambda}_t^{HB}} = \Theta^5 \cdot \overrightarrow{\hat{b}_t^g} - \Theta^6 \cdot \hat{y}_t + \Theta^7 \cdot \hat{\lambda}_t^K, \tag{A.77}$$

where we define $\Theta^5 = \left(\Theta^1\right)^{-1}\Theta^2$, $\Theta^6 = \left(\Theta^1\right)^{-1}\Theta^3$, $\Theta^7 = \left(\Theta^1\right)^{-1}\Theta^4$. Plugging the government's bond portfolio (i.e., (A.49)), the household's loan share (i.e., (A.59)), and the rental price of capital (i.e., (A.61)) into (A.77), we obtain

$$\overrightarrow{\hat{\lambda}_t^{HB}} = \Theta^8 \cdot \hat{b}_t^G - \Theta^6 \cdot \hat{y}_t + \Theta^9 \cdot \left(\hat{z}_t^K + \hat{r}_{t+1}^K\right) - \Theta^{10} \cdot \overrightarrow{\hat{yd}_t} - \Theta^{11} \cdot \mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] - \Theta^{12} \cdot \overrightarrow{\hat{z}_t} + \Theta^{13} \cdot \overrightarrow{\hat{u}_t^B},$$

where we define

$$\Theta^8 = \Theta^5 \cdot \overrightarrow{1_{Fx1}}, \qquad \Theta^{10} = \Theta^7 \cdot \Psi^{13}, \qquad \Theta^{12} = \Theta^7 \cdot \Psi^{12},$$
$$\Theta^9 = \Theta^7 \cdot \kappa^S\left(1-\lambda^K\right), \qquad \Theta^{11} = \Theta^7 \cdot \Psi^{14}, \qquad \Theta^{13} = \Theta^5 \cdot \Xi.$$

By plugging the household's optimal portfolio (i.e., (A.47)) into the above, we obtain

$$\overrightarrow{\hat{yd}_t} = \Theta^{14} \cdot \hat{b}_t^G - \Theta^{15} \cdot \hat{y}_t + \Theta^{16} \cdot \hat{r}_{t+1}^K - \Theta^{17} \cdot \mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] - \Theta^{18} \cdot \overrightarrow{\hat{z}_t} + \Theta^{19} \cdot \hat{z}_t^K + \Theta^{20} \cdot \overrightarrow{\hat{u}_t^B},$$

where $\Theta^{14} = \left[\Theta^{10} + \Psi^2\right]^{-1}\Theta^8, \Theta^{15} = \left[\Theta^{10} + \Psi^2\right]^{-1}\Theta^6, \Theta^{16} = \left[\Theta^{10} + \Psi^2\right]^{-1}\Theta^9, \Theta^{17} = \left[\Theta^{10} + \Psi^2\right]^{-1}\left[\Theta^{11} + \Psi^3\right], \Theta^{18} = \left[\Theta^{10} + \Psi^2\right]^{-1}\Theta^{12}, \Theta^{19} = \left[\Theta^{10} + \Psi^2\right]^{-1}\left[\Theta^9 - \Psi^1\right], \Theta^{20} = \left[\Theta^{10} + \Psi^2\right]^{-1}\Theta^{13}$.

Log-linearization: the Yield-Curve-Control (YCC) policy specific derivations Linearizing the Taylor rule for f-maturity bond (i.e., (24d)) yields[1]

$$\hat{yd}_t^{YCC,f} = \gamma_{CP}^{f} \hat{yd}_t^{CP,f} + \left(1 - \gamma_{CP}^{f}\right) \left[\gamma_{\pi}^{f} \hat{\pi}_t + \tilde{\varepsilon}_t^{YD^f}\right], \quad f \geq 2. \tag{A.78}$$

We define a $(F-1)\times(F-1)$ matrix Γ^{CP} with $\Gamma^{CP}_{ff} = \gamma^{f+1}_{CP}$ for $f = 1 \sim F-1$ and $\Gamma^{CP}_{ij} = 0$ for $i \neq j$. From (36), we define $\mathcal{T}^{YD}_{(f\geq 2)}$, a $(F-1) \times L$ matrix with $\mathcal{T}^{YD}_{(f\geq 2),f,l} = \tau^{YD}_{f+1,l}$ (row f, column l) and the vector of Taylor coefficients $\overrightarrow{\gamma_{\pi}}_{(f\geq 2)} = \left[\gamma_{\pi}^{2}, \ldots, \gamma_{\pi}^{F}\right]'$. If we construct such vectors as

$$\overrightarrow{yd_t^{\hat{Y}CC}}_{(f\geq 2)} = \left[\hat{yd}_t^{YCC,2}, \ldots, \hat{yd}_t^{YCC,F}\right]', \quad \overrightarrow{yd_t^{\hat{C}P}}_{(f\geq 2)} = \left[\hat{yd}_t^{CP,2}, \ldots, \hat{yd}_t^{CP,F}\right]', \tag{A.79}$$

then above equation (A.78) can be written in vector form as

$$\overrightarrow{yd_t^{\hat{Y}CC}}_{(f\geq 2)} = \Gamma^{CP} \overrightarrow{yd_t^{\hat{C}P}}_{(f\geq 2)} + (I - \Gamma^{CP}) \cdot \left[\overrightarrow{\gamma_{\pi}}_{(f\geq 2)} \cdot \hat{\pi}_t + \mathcal{T}^{YD}_{(f\geq 2)} \cdot \overrightarrow{\varepsilon_t^{YD}}\right], \tag{A.80}$$

where I is the identity matrix of size $F-1$. Since $\overrightarrow{yd_t^{\hat{C}P}}$ is the yield vector that prevails in the counterfactual scenario where the current yield is determined by conventional monetary policy, its dynamics will follow

$$\overrightarrow{\hat{yd}_t^{CP}} = \Theta^{14} \cdot \hat{b}_t^G - \Theta^{15} \cdot \hat{y}_t + \Theta^{16} \cdot \hat{r}^K_{t+1} - \Theta^{17} \cdot \mathbb{E}_t \left[\overrightarrow{\hat{yd}_{t+1}^{CP}}\right] - \Theta^{18} \cdot \overrightarrow{\hat{z}_t} + \Theta^{19} \cdot \hat{z}_t^K + \Theta^{20} \cdot \overrightarrow{\hat{u}_t^B},$$

where coefficients Θ^i for $i = 14 \sim 20$ are the same as in the conventional policy case, and $\overrightarrow{yd_t^{\hat{C}P}}$ and $\overrightarrow{yd_t^{\hat{Y}CC}}$ are defined as $\overrightarrow{yd_t^{\hat{Y}CC}} = [\hat{yd}_t^{YCC,1}, \overrightarrow{yd_t^{\hat{Y}CC}}'_{(f\geq 2)}]'$, $\overrightarrow{yd_t^{\hat{C}P}} = [\hat{yd}_t^{YCC,1}, \overrightarrow{yd_t^{\hat{C}P}}'_{(f\geq 2)}]'$. where $\hat{yd}_t^{YCC,1}$ follows the Taylor rules in (24a) and (24b). Now that $\overrightarrow{yd_t^{\hat{Y}CC}}$ governs households' intertemporal decisions, (A.71) becomes

$$\hat{y}_t = \mathbb{E}_t\Big[\hat{y}_{t+1} + \Psi^{46} \cdot \hat{\pi}_{t+1} - \Psi^{47} \cdot \overrightarrow{\hat{z}}_t - \Psi^{48} \cdot \hat{z}_t^K - \Psi^{49} \cdot \overrightarrow{\hat{yd}_t^{YCC}} - \Psi^{50} \cdot \mathbb{E}_t \left[\overrightarrow{\hat{yd}_{t+1}^{YCC}}\right]$$
$$- \Psi^{51} \cdot \hat{r}^K_{t+1} - \Psi^{52} \cdot (\hat{k}_t - \hat{\varepsilon}_t^A) + \Psi^{53} \cdot \hat{k}_{t+1} - \Psi^{54} \cdot \hat{k}_{t+2} + \Psi^{55} \cdot \hat{u}_t^G\Big].$$

[1] If $\rho_1 \neq 0$ or $\rho_2 \neq 0$ in (24d), then the policy rule in (A.78) should account for them and target output as well. We intentionally assume that the policy rule here targets inflation only for simplicity of expressions.

Appendix B Calibration and Estimation Strategy

B.1 Calibrating $\{z^f\}_{f=1}^F$ and z^K at the Steady State

Calibration of $\left\{z^f\right\}_{f=1}^F$ We explain how to calibrate $\left\{z^f\right\}_{f=1}^F$ to match the yield curve. Based on data on yields of bonds with different maturities, we calculate each f-maturity bond's average holding returns $\left\{R^f\right\}$, which we would use as our calibration target.

1. Compute the return ratio $\left\{\frac{R^F}{R^{HB}}\right\}$.[2]

2. Back out steady state bond shares $\left\{\lambda^{HB,f}\right\}$ using equation (A.23)

3. Normalize $z^1 = 1$ and obtain initial guess for $\{z^{j,guess}\}$. Set $z^{j,old} = z^{j,guess}$ in the iteration below.

4. Construct $\tilde{\Phi}^{old}$ using the following formula, where the return ratios $\{\frac{R^f}{R^{HB}}\}$ across maturities are obtained from the data:

$$\tilde{\Phi}^{old} = \left[1 + \sum_{f=2}^{F}\left[z^j\left(\frac{R^f}{R^{HB}}\right)\right]^{\kappa_B}\right]^{\frac{1}{\kappa_B}}.$$

5. Back out new $z^{f,new}$, $f = 2, \ldots, F$ estimates using:

$$z^{f,new} = \left(\lambda^{HB,f}\right)^{\frac{1}{\kappa_B}}\left(\frac{R^f}{R^{HB}}\right)^{-1}\tilde{\Phi}^{old}$$

6. If difference with $\tilde{\Phi}^{old}$ is large, set $z^{f,old} = z^{f,new}$ and start again from the step 4

Calibration of z^K We calibrate z^K such that the model's steady-state return on the household's bond portfolio, R^{HB}, matches with the observed data average. This alignment is achieved by finding the steady-state value of R^K from (A.31),[3] and subsequently replacing the values of R^{HB} and R^K into equation (A.29), which enables us to recover the z^K value consistent with the model's moment.

[2] To determine the return on the household's bond portfolio R^{HB}, we combine the data on the average returns by maturity $\{R^f\}_{f=1}^F$ with the portfolio shares $\{\lambda^{HB,f}\}_{f=1}^F$.

[3] In the context of our present calibration, the derived R^K is situated within the range corresponding to the average corporate debt rate across varied ratings.

B.2 Elasticity Estimation, κ_B

Combining the log-linear approximation of equation (3), we obtain

$$\widehat{\log\left(\lambda_t^{H,f}\right)} = \kappa_B \cdot \left(1 - \lambda^{H,f}\right) \cdot E_t\left[\hat{r}_{t+1}^{f-1}\right] - \kappa_B \cdot \sum_{j \neq f} \lambda^{H,j} \cdot E_t\left[\hat{r}_{t+1}^{j-1}\right] + \varepsilon_t^f \ ,$$

where $\varepsilon_t^f \equiv \kappa_B \cdot \sum_{j \neq f} \lambda_t^{H,j} \cdot \left[\widehat{\log\left(z_t^f\right)} - \widehat{\log\left(z_t^j\right)}\right]$ is a residual term containing the effects on the households' bond portfolio of shocks to different maturity preferences along the yield curve. Differentiation across maturities yields the following expression

$$\log\left(\lambda_t^{H,f}\right) - \log\left(\lambda_t^{H,l}\right) = \alpha^{fl} + \kappa_B \cdot E_t\left[r_{t+1}^{f-1} - r_{t+1}^{l-1}\right] + \varepsilon_t^{fl} \ , \tag{B.1}$$

where α^{fl} denotes a constant, and $\varepsilon_t^{fl} = \varepsilon_t^f - \varepsilon_t^l$ represents a residual term embodying the discrepancy between the preference shocks for bond maturities f and l. The expected bond return spread, denoted as $E_t\left[r_{t+1}^{f-1} - r_{t+1}^{l-1}\right]$, poses challenges for empirical observation. Therefore, we turn to the following approximation centered on the bond yield spread:

$$\begin{aligned} E_t\left[r_{t+1}^{f-1} - r_{t+1}^{l-1}\right] =& yd_t^f - yd_t^l \\ & - (f-1) \cdot \underbrace{E_t\left[yd_{t+1}^f - yd_t^f\right]}_{\text{add as control}} + (l-1) \cdot \underbrace{E_t\left[yd_{t+1}^l - yd_t^l\right]}_{\text{add as control}} \\ & + (f-1) \cdot \underbrace{E_t\left[yd_{t+1}^f - yd_{t+1}^{f-1}\right]}_{\approx 0 \text{ (by assumption)}} - (l-1) \cdot \underbrace{E_t\left[yd_{t+1}^l - yd_{t+1}^{l-1}\right]}_{\approx 0 \text{ (by assumption)}} \ , \end{aligned}$$

where the spread $yd_t^f - yd_t^l$ in the initial line is directly observable from the data, the expected yield change in the subsequent line can be approximated by employing the realized changes as control variables, and the terms in the final line may be assumed to be close to zero over short time intervals. The final equation used for the empirical estimation of the elasticity parameter κ_B becomes:

$$\log\left(\lambda_{t+h}^{H,f}\right) - \log\left(\lambda_{t+h}^{H,l}\right) = \alpha_h^{fl} + \kappa_{B,h} \cdot \left[yd_t^f - yd_t^l\right] + \mathbf{x_t}'\beta_{\mathbf{h}}^{\mathbf{fl}} + \varepsilon_{t+h}^{fl}, \ h \geq 0 \ , \tag{B.2}$$

where $\mathbf{x_t}$ denotes a vector of control variables accompanied by the corresponding coefficients $\beta_{\mathbf{h}}^{\mathbf{fl}}$, and the time subindex $h \geq 0$ accommodates a lagged effect from the yield spread to the bond portfolio composition which might occur in practice. Equation (B.2) is estimated across various horizons h utilizing Jordà local projection methods, thereby facilitating the determination of a plausible range for the elasticity parameter κ_B.

The unbiased estimation of κ_B in equation (B.2) requires the fluctuations in the bond yield spread $yd_t^f - yd_t^l$ to be uncorrelated with shocks in the relative preferences for each maturity, as captured by ε_{t+h}^{fl}. This implies that any other aggregate shocks, uncorrelated to the contemporaneous (and/or future) maturity preferences of households, could serve as potentially valid instruments for the yield spread. Following this rationale, we instrument the changes in the contemporaneous yield spread with its own lagged value, while at the same time, we incorporate the lags of the dependent variable as controls to eliminate any potential serial correlation of the preference shocks. The dependent variable is

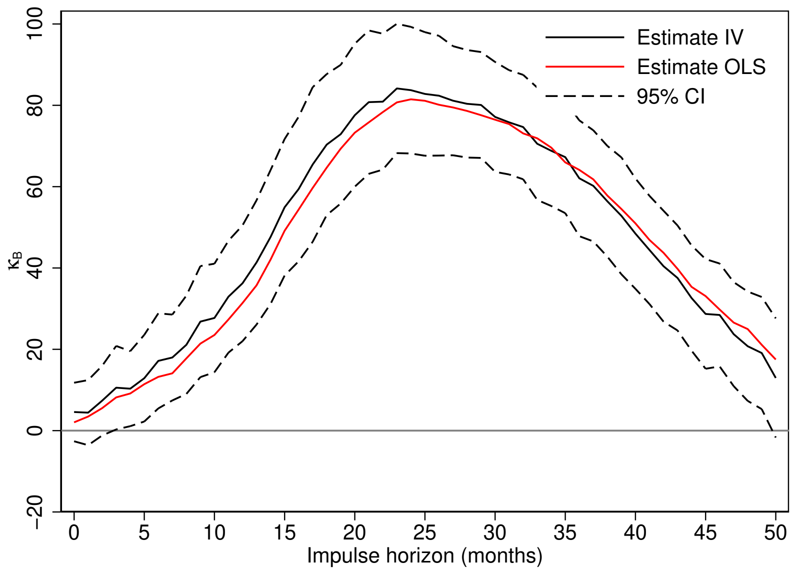


Figure B.1: Impulse-Response to a shock in the yield spread, $yd_t^f - yd_t^l$. The figure presents the coefficient estimates for the bond portfolio elasticity, κ_B, in equation (B.2), following the estimation methodology detailed in appendix B.2. The solid black line illustrates the estimate from the instrumental variables (IV) regression, with dashed lines indicating the 95% robust confidence intervals. The red line exhibits alternative OLS estimates. The sample period is from 2003m3 to 2019m3.

defined as the log-difference between the aggregate household portfolio shares in a group of long-maturity bonds and a group of short-maturity bonds, respectively. For the long-maturity group, we calculate the share of bonds with maturities ranging from 5 to 10 years within the households' portfolio, whereas for the short-maturity group, bonds with maturities spanning from 15 to 90 days are considered. For both groups, the aggregate household portfolio holdings are computed on a monthly basis by deducting the U.S. Treasury securities held by the Federal Reserve from the Government's outstanding Treasury amounts within the selected maturity ranges.[4] The principal regressor employed is the spread between the market yields of the 7-year and the 1-month constant maturity U.S. Treasury securities, which lie within the maturity bands of the selected portfolio shares in the dependent variable. Additionally, we control for the one-month-ahead changes in the 7-year and the 1-month constant maturity yields, along with the first three lags of the dependent variable. The regressions are estimated across the sample period extending from 2003m3 to 2019m3.[5]

Figure B.1 delineates the IV and OLS estimates derived from Jordà local projections across a fifty-month horizon, accompanied by the 95% robust confidence bands pertinent to the primary IV regression. Both estimates largely align with the anticipated reaction of aggregate household portfolio shares, as posited by the model, in response to a shock to the yield spread. For calibration purposes, we select a value of $\kappa_B = 10$, which is consistent with the short-term portfolio response observed in the Figure B.1.

[4]The outstanding amounts of Government Treasuries are reported in the U.S. Treasury Monthly Statement of the Public Debt (MSPD).

[5]The lower bound of the sample period is dictated by the availability of maturity-disaggregated statistics concerning the Federal Reserve's Treasury bond portfolio, commencing from 2002m12.

Appendix C Welfare

C.1 Deriving a second-order welfare approximation

In order to approximate welfare up to a second-order, we cannot discard $\hat{\Delta}_t$, which is the price dispersion's log-deviation from its steady-state value in the presence of trend inflation.

Step 1: For any variable X, we define $\bar{X}$ as its steady-state value (with the positive trend inflation $\bar{\Pi} > 1$) and $\bar{X}^F$ as its flexible price steady-state value. Also define (small) letter $\tilde{x}$ as log-deviation of X around $\bar{X}^F$, and $\hat{x}$ as log-deviation of X around $\bar{X}$.

Constrained efficient (i.e., flexible-price) steady state With the optimal production subsidy $\zeta^F = (\epsilon - 1)^{-1}$ that eliminates the monopolistic competition distortion, there is no distortion other than the firms' financing constraint in the flexible-price steady state economy anymore.[6] In particular, each firm's price resetting condition (i.e., (A.2)) becomes

$$1 = \frac{P_t^*}{P_t} = \underbrace{\frac{(1+\zeta^F)^{-1}\epsilon}{\epsilon - 1}}_{=1} \cdot \frac{MC_t}{P_t} = \frac{MC_t}{P_t}, \tag{C.1}$$

where we use the fact that all firms become identical, and thus $MC_t(\nu) = MC_t$ for all $\nu \in [0, 1]$. Therefore, the real marginal cost becomes 1 for all firms. Plugging the unit real marginal cost (i.e., (A.3)) into the individual firm's labor demand (i.e., (16)) with $W_t(\nu) = W_t$ for $\forall \nu$, and defining $n_t = \frac{N_t}{\bar{N}_t}$ and $y_t = \frac{Y_t}{A_t \bar{N}_t}$, we obtain

$$n_t = (1-\alpha) y_t \left(\frac{P_t^K}{P_t}\right)^{\alpha} \left(\frac{W_t}{P_t A_t}\right)^{-\alpha} = (1-\alpha) y_t \left(\frac{W_t}{P_t A_t}\right)^{-1}, \tag{C.2}$$

which, with the household's intra-temporal consumption-labor decision (i.e., (9)), becomes:

$$\frac{n_t^{\frac{1}{\eta}}}{c_t^{-1}} = (1-\alpha)\frac{y_t}{n_t}, \tag{C.3}$$

which is the social efficiency condition that ensures that the household's marginal rate of substitution matches with the marginal rate of technical substitution. Therefore, at the

[6] The capital producing firm is competitive and thus our economy features no friction other than the firms' financing constraint if it were not nominal rigidity nor trend inflation. Still, the constraint on loan issuance does not affect firms' marginal decisions on labor and capital.

flexible-price steady state, the new constant Φ, which will turn out to enter in the per-period welfare later, can be calculated as

$$\Phi \equiv (\bar{n}^F)^{1+\frac{1}{\eta}} = (1-\alpha)\frac{\bar{y}^F}{\bar{c}^F} = (1-\alpha)\frac{\bar{Y}^F}{\bar{C}^F}, \tag{C.4}$$

where $\bar{n}^F$, $\bar{y}^F$, and $\bar{c}^F$ are values of normalized labor, output, and consumption, respectively.

Step 2: With aggregation equations (A.5) and (A.4), we obtain

$$\left(\frac{N_t}{\bar{N}_t}\right)^{1-\alpha}\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right)^{\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(GA_t\cdot GN)^{\alpha}\left(\frac{Y_t}{A_t\bar{N}_t}\right)\Delta_t^{(1-\alpha)\left[\frac{\eta}{\eta+1}+\frac{\alpha}{1-\alpha}\right]}, \tag{C.5}$$

which is the aggregate production function with the price dispersion Δ_t. Plugging steady-state (with trend-inflation) capital (i.e., A.37)) and output (i.e., (A.39)) into (C.5) yields

$$\frac{N}{\bar{N}} = \left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}(GA\cdot GN)^{\alpha}\left(\xi^K\right)^{-\alpha}\right]^{\frac{1}{1-\alpha}}\Delta^{\frac{\eta+\alpha}{(\eta+1)(1-\alpha)}}\left(\xi^Y\right)^{\frac{1}{1-\alpha}\left(\frac{\eta+\alpha}{\eta+1}\right)}\left(\xi^C\right)^{-\frac{\eta}{\eta+1}} \tag{C.6}$$

where ξ^K in (A.37), ξ^Y in (A.39), and ξ^C in (A.38) all depend on θ and the trend inflation Π. Therefore, we see that $\bar{n} \neq \bar{n}^F$ and define $\log X_n \equiv \log\bar{n} - \log\bar{n}^F$, which will turn out to be useful later when we calculate the household's first-order labor cost.

Step 3: Price dispersion with positive trend inflation

Delta method Before we start, we would use this approximation throughout this section. For a random variable X with $E(X) = \mu_X$, we have

$$\mathrm{Var}\left(f(X)\right) = f'(\mu_X)^2\cdot\mathrm{Var}(X) + \text{h.o.t.} \tag{C.7}$$

Price dispersion We use lower-case p_t and $p_t(\nu)$ as logarithms of P_t and $P_t(\nu)$. By applying the above delta method to $P_t^{1-\epsilon} = \mathbb{E}_{\nu}\left(P_t(\nu)^{1-\epsilon}\right)$, we obtain

$$p_t = \underbrace{\int_0^1 p_t(\nu)d\nu}_{\equiv\bar{p}_t} + \frac{1}{2}\left(\frac{1}{1-\epsilon}\right)\frac{\mathrm{Var}_{\nu}\left(P_t(\nu)^{1-\epsilon}\right)}{\mathbb{E}_{\nu}\left(P_t(\nu)^{1-\epsilon}\right)^2} + \text{h.o.t.} \tag{C.8}$$

where we define $\bar{p}_t \equiv \mathbb{E}_\nu(p_t(\nu))$. Applying the delta method to $\mathrm{Var}_\nu(P_t(\nu)^{1-\epsilon})$, we have

$$\mathrm{Var}_\nu\left(P_t(\nu)^{1-\epsilon}\right) = (1-\epsilon)^2 \cdot \left[\exp((1-\epsilon)\bar{p}_t)\right]^2 \cdot \mathrm{Var}_\nu(p_t(\nu)), \tag{C.9}$$

where we define $D_t \equiv \mathrm{Var}_\nu(p_t(\nu))$. Applying the delta method to $\mathbb{E}_\nu(P_t(\nu)^{1-\epsilon})$, we obtain

$$\mathbb{E}_\nu\left(P_t(\nu)^{1-\epsilon}\right) = \exp((1-\epsilon)\bar{p}_t)\left[1 + \frac{(1-\epsilon)^2}{2}D_t\right]. \tag{C.10}$$

Plugging (C.9) and (C.10) into (C.8), we obtain

$$p_t = \bar{p}_t + \frac{1-\epsilon}{2} \cdot \frac{D_t}{\left[1 + \frac{(1-\epsilon)^2}{2}D_t\right]^2}, \tag{C.11}$$

which we linear-approximate around $D_t = \bar{D}$ and obtain

$$p_t - \bar{p}_t = \underbrace{\frac{1-\epsilon}{2} \cdot \frac{\bar{D}}{\left[1 + \frac{(1-\epsilon)^2}{2}\bar{D}\right]^2}}_{\equiv \Theta_1^p} + \underbrace{\frac{1-\epsilon}{2} \cdot \frac{1 - \frac{(1-\epsilon)^2}{2}\bar{D}}{\left[1 + \frac{(1-\epsilon)^2}{2}\bar{D}\right]^3}}_{\equiv \Theta_2^p} \cdot (D_t - \bar{D}) = \Theta_1^p + \Theta_2^p(D_t - \bar{D}). \tag{C.12}$$

Now from our original definition of the price dispersion Δ_t (i.e., (28)), we take logarithm on both sides, linear-approximate around $\bar{D}$, and plug (C.12) into it to attain

$$\begin{aligned}
\ln \Delta_t &= \ln \int_0^1 \left(\frac{P_t(\nu)}{P_t}\right)^{\frac{-\epsilon(\eta+1)}{\eta+\alpha}} d\nu \\
&= \frac{\epsilon(\eta+1)}{\eta+\alpha}(p_t - \bar{p}_t) + \ln\left(1 + \frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2 \bar{D}\right) + \frac{\frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2}{1 + \frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2 \bar{D}}(D_t - \bar{D}) \\
&= \Theta_1^\Delta + \Theta_2^\Delta \cdot (D_t - \bar{D}) + \text{h.o.t},
\end{aligned}$$

where

$$\Theta_1^\Delta \equiv \frac{\epsilon(\eta+1)}{\eta+\alpha} \cdot \frac{1-\epsilon}{2} \cdot \frac{\bar{D}}{\left[1 + \frac{(1-\epsilon)^2}{2}\bar{D}\right]^2} + \ln\left(1 + \frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2 \bar{D}\right), \tag{C.13}$$

$$\Theta_2^\Delta \equiv \frac{\epsilon(\eta+1)}{\eta+\alpha} \cdot \frac{1-\epsilon}{2} \cdot \frac{1 - \frac{(1-\epsilon)^2}{2}\bar{D}}{\left[1 + \frac{(1-\epsilon)^2}{2}\bar{D}\right]^3} + \frac{\frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2}{1 + \frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2 \bar{D}}. \tag{C.14}$$

If we define b_t as the logarithm of the newly price-resetting firm's relative price P_t^*/P_t and $\bar{b}$ as its steady state value, we have $\bar{b} \neq 0$ due to the trend inflation. Combining (A.8) and

(A.10) and linearizing, we obtain

$$b_t \equiv p_t^* - p_t = \bar{b} + \underbrace{\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}}_{\equiv M}\hat{\pi}_t = \bar{b} + M\cdot\hat{\pi}_t, \text{ with } \bar{b} = \frac{1}{\epsilon-1}\ln\left(\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right). \tag{C.15}$$

With $D_t = \mathrm{Var}_\nu(p_t(\nu)) = \mathbb{E}_\nu((p_t(\nu) - p_t + p_t - \bar{p}_t)^2)$, we can write it as

$$\begin{aligned} D_t &= \int_0^{1-\theta}(p_t^* - p_t)^2 d\nu + 2\left(\int_0^{1-\theta}(p_t^* - p_t)d\nu\right)(p_t - \bar{p}_t) + (1-\theta)(p_t - \bar{p}_t)^2 + \int_{1-\theta}^1 (p_{t-1}(\nu) - \bar{p}_t)^2 d\nu \\ &= (1-\theta)(p_t^* - p_t)^2 + 2(1-\theta)(p_t^* - p_t)(p_t - \bar{p}_t) + (1-\theta)(p_t - \bar{p}_t)^2 + \theta D_{t-1} + \theta(\bar{p}_t - \bar{p}_{t-1})^2, \end{aligned} \tag{C.16}$$

where we use

$$\int_{1-\theta}^1 (p_{t-1}(\nu) - \bar{p}_t)^2 d\nu = \theta D_{t-1} + \theta(\bar{p}_{t-1} - \bar{p}_t)^2. \tag{C.17}$$

Conjecture Following Coibion et al. (2012), we conjecture the dynamics of D_t up to a second-order as[7]

$$D_t - \bar{D} = \kappa_D\hat{\pi}_t + Z_D(\hat{\pi}_t)^2 + F_D(D_{t-1} - \bar{D}) + G_D(D_{t-1} - \bar{D})\hat{\pi}_t + H_D(D_{t-1} - \bar{D})^2. \tag{C.18}$$

with

$$\begin{aligned} \bar{D} &= (\bar{b} + \Theta_1^p)^2 + \frac{\theta}{1-\theta}(\bar{\pi})^2 \quad \text{(i.e., steady state value of } D_t\text{)}, \\ \kappa_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[2(1-\theta)M(\bar{b} + \Theta_1^p) + 2\theta\bar{\pi}\right], \\ Z_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[(1-\theta)M^2 + 2(1-\theta)M\Theta_2^p\kappa_D + (\Theta_2^p)^2(\kappa_D)^2 + \theta - 2\theta\Theta_2^p\kappa_D\right], \\ F_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[\theta + 2\theta\Theta_2^p\bar{\pi}\right], \\ G_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[2(1-\theta)M\Theta_2^pF_D + 2(\Theta_2^p)^2\kappa_DF_D - 2\theta\Theta_2^pF_D \right. \\ &\qquad \left. + 2\theta\Theta_2^p - 2\theta(\Theta_2^p)^2\kappa_D\right], \\ H_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[(\Theta_2^p)^2(F_D)^2 + \theta(\Theta_2^p)^2 - 2\theta(\Theta_2^p)^2F_D\right]. \end{aligned} \tag{C.19}$$

With no trend inflation, we would have $\pi = 0$ and $\bar{D} = 0$, thus D_t becomes the second-order variable around 0 and we would have $\kappa_D = 0$. However with steady-state inflation $\pi > 0$ and the price dispersion measure $\bar{D} > 0$, as we see in (C.18), D_t includes $\hat{\pi}_t$ term

[7] Following Coibion et al. (2012), we assume κ_D is of the same order as the shock processes, so that the first term becomes of a second-order. Then our log-linearized model derivation without price dispersion term is valid.

as one of its components, with κ_D being of the first-order. Our objective is to derive (C.18) from firms' optimal pricing behaviors and the price dispersion's effects on the aggregate price itself. Plugging (C.12) and (C.15) into (C.16) and replace $(D_t - \bar{D})$ with the conjectured form in (C.18) up to a second-order,[8] and comparing coefficients, we obtain the set of coefficients in (C.19).

Consumption utility We can second-order approximate the utility of consumption as

$$u(c_t) = \log c_t = u(\bar{c}^F) + u'_{\bar{c}^F} \cdot \bar{c}^F \cdot \underbrace{\left(\frac{c_t - \bar{c}^F}{\bar{c}^F}\right)}_{=\tilde{c}_t + \cancel{\frac{1}{2}(\tilde{c}_t)^2}} + \frac{1}{2} u''_{\bar{c}^F} \cdot (\bar{c}^F)^2 \cdot \underbrace{\left(\frac{c_t - \bar{c}^F}{\bar{c}^F}\right)^2}_{=\cancel{(\tilde{c}_t)^2}} + \text{h.o.t} = u(\bar{c}^F) + \tilde{c}_t + \text{h.o.t.} \tag{C.20}$$

<u>Step 4</u>: Labor aggregation and cost

By applying the delta method (i.e., (C.7)) to the labor aggregator, we can obtain[9]

$$\tilde{n}_t - \mathbb{E}_\nu(\tilde{n}_t(\nu)) = \underbrace{\frac{\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\overline{\nabla}}{1 + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)^2 \overline{\nabla}}}_{\equiv \Theta_1^n} + \underbrace{\frac{1}{2}\left(\frac{\eta+1}{\eta}\right) \frac{1 - \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)^2 \overline{\nabla}}{\left[1 + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)^2 \overline{\nabla}\right]^3}}_{\equiv \Theta_2^n} \cdot (\nabla_t - \overline{\nabla}) \tag{C.21}$$

where $\nabla_t \equiv \text{Var}_\nu(\log n_t(\nu))$. The second-order approximation to the firm ν-specific labor cost around the flexible-price steady state yields

$$\frac{\eta}{\eta+1}\left(\frac{N_t(\nu)}{\bar{N}_t}\right)^{\frac{\eta+1}{\eta}} = \frac{\eta}{\eta+1}(\bar{n}^F)^{\frac{\eta+1}{\eta}} + \Phi\left[\tilde{n}_t(\nu) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\tilde{n}_t(\nu)^2\right] + h.o.t \tag{C.22}$$

where the constant Φ is from (C.4). Aggregating (C.22) over firms $\nu \in [0, 1]$ and plugging (C.21) results in

$$\begin{aligned}
\frac{\eta}{\eta+1}\int_0^1 \left(\frac{N_t}{\bar{N}_t}\right)^{\frac{\eta+1}{\eta}} d\nu - \frac{\eta}{\eta+1}(\bar{n}^F)^{\frac{\eta+1}{\eta}} &= \Phi\left[\mathbb{E}_\nu(\tilde{n}_t(\nu)) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\int_0^1 \tilde{n}_t(\nu)^2 d\nu\right] \\
&= -\Phi\left(\Theta_1^n - \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_1^n)^2\right) + \Phi\Bigg[\left(1 - \left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\tilde{n}_t + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\tilde{n}_t^2 \\
&\quad + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_2^n)^2\left(\text{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right)^2 - \frac{\eta+1}{\eta}\Theta_2^n \tilde{n}_t\left(\text{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right) \\
&\quad + \left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1 + 2\Theta_1^n\Theta_2^n) - \Theta_2^n\right)\left(\text{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\overline{\nabla}\Bigg].
\end{aligned} \tag{C.23}$$

[8] In the right-hand side of the expression, $(p_t - \bar{p}_t)^2$ appears and has a second-order term $(D_t - \bar{D})^2$ from (C.12), and we use (C.18) to replace this term with terms related to $(\hat{\pi}_t)^2$, $(D_{t-1} - \bar{D})^2$, and $\hat{\pi}_t(D_{t-1} - \bar{D})$.

[9] In the flexible-price steady-state, there is no heterogeneity among firms, i.e., $\bar{n}^F(\nu) = \bar{n}^F$ for $\forall \nu$.

Labor dispersion From individual firm's labor and capital demand (i.e., (16)) and the household's intra-marginal condition (i.e., (9)), we obtain

$$\tilde{k}_t(\nu) = \left(1 + \frac{1}{\eta}\right) \tilde{n}_t(\nu) + \text{aggregate}, \tag{C.24}$$

where 'aggregate' stands for aggregate variables. Therefore, we obtain

$$\tilde{y}_t(\nu) = \left(1 + \frac{\alpha}{\eta}\right) \tilde{n}_t(\nu) + \text{aggregate}, \tag{C.25}$$

by plugging (C.24) into each firm's production function $\tilde{y}_t(\nu) = \alpha\tilde{k}_t(\nu) + (1-\alpha)\tilde{n}_t(\nu)$. From the Dixit-Stiglitz good demand (i.e., (13)) and with (C.25), we can get

$$\mathrm{Var}_\nu(\tilde{n}_t(\nu)) = \left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2 \mathrm{Var}_\nu(p_t(\nu)), \text{ with } \overline{\nabla} = \left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2 \bar{D}. \tag{C.26}$$

Step 5: Constructing a welfare function: Combining the consumption utility (i.e., (C.20)) and the labor disutility (i.e., (C.23)), we can construct welfare as

$$\begin{aligned}\mathbb{E}U_t - \bar{U}^F = \mathbb{E}\Bigg[&\tilde{c}_t + \Phi\left(\Theta_1^n - \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_1^n)^2\right) - \Phi\Bigg\{\left(1 - \left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\tilde{n}_t + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\tilde{n}_t^2 \\ &+ \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_2^n)^2\left(\mathrm{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right)^2 - \frac{\eta+1}{\eta}\Theta_2^n\tilde{n}_t\left(\mathrm{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right) \\ &+ \left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1 + 2\Theta_1^n\Theta_2^n) - \Theta_2^n\right)\left(\mathrm{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\overline{\nabla}\Bigg\}\Bigg],\end{aligned} \tag{C.27}$$

with from (A.38), (A.39), and (C.6) the flexible-price steady-state utility given as

$$\begin{aligned}\bar{U}^F &= \bar{c}^F - \frac{\eta}{\eta+1}\left(\frac{N^F}{\bar{N}}\right)^{\frac{\eta+1}{\eta}} \\ &= \frac{1}{\eta+1}\left[\log\left(\xi^{C,f}\right) + \frac{\eta+\alpha}{1-\alpha}\log\left(\xi^{Y,f}\right)\right] - \frac{\eta}{\eta+1}\left[\alpha^\alpha(1-\alpha)^{1-\alpha}(GA\cdot GN)^\alpha\left(\xi^{K,f}\right)^{-\alpha}\right]^{\frac{\eta+1}{(1-\alpha)\eta}}\left(\xi^{Y,f}\right)^{\frac{\eta+\alpha}{(1-\alpha)\eta}}\left(\xi^{C,f}\right)^{-1},\end{aligned}$$

where

$$\begin{aligned}\xi^{F,f} &= (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\left[\beta^{-1}\cdot GA\cdot GN - (1-\delta)\right]^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}, \quad \xi^{H,f} = 1 - \gamma_L\left(\frac{R^K}{R^S} - 1\right), \quad \xi^{Y,f} = \frac{\xi^{H,f}}{\xi^{F,f}}, \\ \xi^{K,f} &= \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\cdot GA\cdot GN\cdot\left[\beta^{-1}\cdot GA\cdot GN - (1-\delta)\right]^{-\frac{\eta(1-\alpha)}{\eta+\alpha}}\xi^{Y,f}, \\ \xi^{C,f} &= 1 - \zeta^G - \xi^{K,f}\left(1 - \frac{1-\delta}{GA\cdot GN}\right).\end{aligned}$$

$\xi^{C,f}$, $\xi^{Y,f}$, and $\xi^{K,f}$ are levels of ξ^C, ξ^Y, and ξ^K when $\theta = 0$ (i.e., flexible price steady state). With

$$\log \frac{\xi^Y}{\xi^{Y,f}} = \log\left(\frac{1-\theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}}{1-\theta\beta\Pi^{\epsilon-1}}\right) + \frac{1}{\epsilon-1}\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\log\left(\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right), \quad \text{(C.28)}$$

and

$$\log \frac{\xi^K}{\xi^{K,f}} = \log\left(\frac{\xi^Y}{\xi^{Y,f}}\right) + \log\Delta, \quad \text{(C.29)}$$

$$\log \frac{\xi^C}{\xi^{C,f}} = \log \frac{1-\zeta^G-\xi^K\left(1-\dfrac{1-\delta}{GA\cdot GN}\right)}{1-\zeta^G-\xi^{K,f}\left(1-\dfrac{1-\delta}{GA\cdot GN}\right)}, \quad \text{(C.30)}$$

where Δ at the steady state with trend inflation is defined in (A.32), which gives

$$\log\Delta = \log\left(\frac{1-\theta}{1-\theta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}}\right) + \frac{\epsilon}{\epsilon-1}\left(\frac{\eta+1}{\eta+\alpha}\right)\log\left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta}\right). \quad \text{(C.31)}$$

If we define $\log X_c = \tilde{c}_t - \hat{c}_t$ as the log-difference in consumption between our steady state (with trend-inflation) and the flexible price steady state, we obtain

$$\log X_c \equiv \bar{c} - \bar{c}^F = \frac{1}{\eta+1}\left[\log\left(\frac{\xi^C}{\xi^{C,f}}\right) + \frac{\eta+\alpha}{1-\alpha}\log\left(\frac{\xi^Y}{\xi^{Y,f}}\right)\right]. \quad \text{(C.32)}$$

For labor, we define $\log X_n$ as the log-difference in labor between our steady state with trend inflation and the flexible price steady state, which is, with the help of (C.6), given by

$$\log X_n \equiv \bar{n} - \bar{n}^F = -\frac{\alpha}{1-\alpha}\log\frac{\xi^K}{\xi^{K,f}} + \frac{1}{1-\alpha}\left(\frac{\eta+\alpha}{\eta+1}\right)\log\frac{\xi^Y}{\xi^{Y,f}} - \frac{\eta}{\eta+1}\log\frac{\xi^C}{\xi^{C,f}} + \frac{\eta+\alpha}{(\eta+1)(1-\alpha)}\log\Delta. \quad \text{(C.33)}$$

With $\tilde{c}_t = \hat{c}_t + \log X_c$, $\tilde{n}_t = \hat{n}_t + \log X_n$, and the stationarity assumption (following Coibion et al. (2012)), we can get

$$\mathbb{E}\left[\tilde{c}_t - \Phi\left(1-\left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\tilde{n}_t\right] = \log X_c - \Phi\left(1-\left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\log X_n. \quad \text{(C.34)}$$

Second order terms: With $\tilde{n}_t = \hat{n}_t + \log X_n$, second-order terms can be collected as

$$-\Phi\left[\frac{\eta+1}{2\eta}\mathbb{E}\left(\hat{n}_t^2\right)+\frac{\eta+1}{2\eta}(\Theta_2^n)^2\mathbb{E}\left(\left(\mathrm{Var}_\nu(\tilde{n}_t(\nu))-\overline{\nabla}\right)^2\right)-\frac{\eta+1}{\eta}\Theta_2^n\mathbb{E}\left(\hat{n}_t\left(\mathrm{Var}_\nu(\hat{n}_t(\nu))-\overline{\nabla}\right)\right)\right.$$
$$\left.+\left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1+2\Theta_1^n\Theta_2^n)-\Theta_2^n-\frac{\eta+1}{\eta}\Theta_2^n\log X_n\right)\mathbb{E}\left(\mathrm{Var}_\nu(\hat{n}_t(\nu))-\overline{\nabla}\right)\right]\Bigg], \tag{C.35}$$

which, after we can plug (C.26) into, becomes

$$-\Phi\left[\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\mathrm{Var}\left(\hat{n}_t\right)+\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_2^n)^2\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^4\mathbb{E}(D_t-\bar{D})^2\right.$$
$$-\frac{\eta+1}{\eta}\Theta_2^n\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2\mathrm{Cov}(\hat{n}_t,D_t)$$
$$\left.+\left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1+2\Theta_1^n\Theta_2^n)-\Theta_2^n\left(1+\frac{\eta+1}{\eta}\log X_n\right)\right)\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2\mathbb{E}(D_t-\bar{D})\right]\Bigg]. \tag{C.36}$$

Finally, by plugging (C.18) into (C.36), we get the following proposition. Sine κ_D is of the same order as shock processes, up to a second-order, we can ignore covariance terms and the square term of D_t. Therefore, a 2^{nd}-order approximation to the expected per-period welfare would be given as

$$\mathbb{E}U_t-\bar{U}^F=\Omega_0+\Omega_n\mathrm{Var}(\hat{n}_t)+\Omega_\pi\mathrm{Var}(\hat{\pi}_t), \tag{C.37}$$

with

$$\begin{aligned}
\Omega_0 &= \log X_c-\Phi\left(1-\left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\log X_n+\Phi\left(\Theta_1^n-\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_1^n)^2\right)-\Phi\frac{1}{2}\frac{\eta+1}{\eta}(\log X_n)^2\\
&\quad-\Phi\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2\bar{D},\\
\Omega_n &= -\Phi\frac{1}{2}\left(\frac{\eta+1}{\eta}\right),\\
\Omega_\pi &= -\Phi\left[\left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1+2\Theta_1^n\Theta_2^n)-\Theta_2^n\left(1+\frac{\eta+1}{\eta}\log X_n\right)\right)\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2\frac{Z_D}{1-F_D}\right],
\end{aligned} \tag{C.38}$$

where $\log X_c$ and $\log X_n$ are defined in (C.32) and (C.33) respectively, coefficients Θ_1^n, Θ_2^n are given in equation (C.21), and $\bar{D}$ is given by jointly solving (C.12) (i.e., definition of Θ_1^p) and (C.19). $\kappa_D, Z_D, F_D, G_D, H_D$ are given in (C.19).

<u>Supplementary Material</u>: Not for Publication

1 Summary of Equilibrium Equations

1.1 Equilibrium Equations: Conventional Policy (CP)

$$(i).\ \frac{C_t}{A_t\bar{N}_t} = \left(1-\zeta_t^G\right)\left(\frac{Y_t}{A_t\bar{N}_t}\right) + \left(\frac{1-\delta}{GA_t\cdot GN}\right)\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right) - \left(\frac{K_{t+1}}{A_t\bar{N}_t}\right) \tag{1.1}$$

$$(ii).\ 1 = \beta\cdot\mathbb{E}_t\left[\frac{R^S_{t+1}}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\right] \tag{1.2}$$

$$(iii).\ \lambda_t^{HB,1} = 1-\sum_{f=2}^{F}\lambda_t^{HB,f} \tag{1.3}$$

$$(iv).\ -\left(\frac{B_t^{G,f}}{A_t\bar{N}_tP_t}+\frac{\overline{B^{CB,f}}}{A\bar{N}P}\right)\cdot\left(\lambda_t^{HB,f}\right)^{-1} = \gamma_L\cdot\left(1+\zeta^F\right)\cdot\left(\frac{1-\lambda_t^K}{\lambda_t^K}\right)\left(\frac{Y_t}{A_t\bar{N}_t}\right),\quad\forall f>1 \tag{1.4}$$

$$(v).\ YD_t^1 = \max\left\{YD_t^{1*},1\right\} \tag{1.5}$$

$$(vi).\ YD_t^{1*} = \overline{YD}^1\cdot\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_y}\cdot\exp\left(\tilde{\varepsilon}_t^{YD^1}\right) \tag{1.6}$$

$$(vii).\ \lambda_t^{HB,f} = \left(\frac{\mathbb{E}_t\left[\frac{\beta z_t^f}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\cdot\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\frac{\left(YD_{t+1}^{f-1}\right)^{-(f-1)}}{\left(YD_t^f\right)^{-f}}\right]}{\Phi_t^B}\right)^{\kappa_B},\quad\forall f \tag{1.7}$$

$$(viii).\ \Phi_t^B = \left[\sum_{j=1}^{F}\mathbb{E}_t\left[\frac{\beta z_t^j}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\cdot\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\frac{\left(YD_{t+1}^{j-1}\right)^{-(j-1)}}{\left(YD_t^j\right)^{-j}}\right]^{\kappa_B}\right]^{\frac{1}{\kappa_B}} \tag{1.8}$$

$$(ix).\ \lambda_t^K = \left(\frac{z_t^K\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]}{\Phi_t^S}\right)^{\kappa_S} \tag{1.9}$$

$$(x).\ \Phi_t^S = \left[\left(\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{HB}\right]\right)^{\kappa_S}+\left(z_t^K\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]\right)^{\kappa_S}\right]^{\frac{1}{\kappa_S}} \tag{1.10}$$

$$(xi).\;\; R_t^j = \sum_{f=0}^{F-1} \lambda_{t-1}^{j,f+1} \frac{\left(YD_t^f\right)^{-f}}{\left(YD_{t-1}^{f+1}\right)^{-(f+1)}},\;\; j \in \{HB, G, CB\} \tag{1.11}$$

$$(xii).\;\; R_t^S = \left(1-\lambda_{t-1}^K\right) R_t^{HB} + \lambda_{t-1}^K R_t^K \tag{1.12}$$

$$(xiii).\;\; 1 = \mathbb{E}_t \left[Q_{t,t+1}\Pi_{t+1} \left[(1-\delta) + \frac{P_{t+1}^K}{P_{t+1}} \right] \right] \tag{1.13}$$

$$(xiv).\;\; F_t = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1} \right) \left(\frac{C_t}{A_t\bar{N}_t} \right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t\bar{N}_t} \right)^{\frac{\eta+1}{\eta+\alpha}} \left(\frac{P_t^K}{P_t} \right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta\mathbb{E}_t \left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} F_{t+1} \right] \tag{1.14}$$

$$(xv).\;\; H_t = \left(\frac{C_t}{A_t\bar{N}_t} \right)^{-1} \frac{Y_t}{A_t\bar{N}_t} \left[1 - \gamma_L \cdot \left(\widetilde{R}_{t+1}^K - 1 \right) \right] + \theta\beta\mathbb{E}_t \left[\Pi_{t+1}^{\epsilon-1} H_{t+1} \right] \tag{1.15}$$

$$(xvi).\;\; \frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}} \right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} \tag{1.16}$$

$$(xvii).\;\; \Delta_t = (1-\theta) \left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta} \right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \Delta_{t-1} \tag{1.17}$$

$$(xviii).\;\; \frac{N_t}{\bar{N}_t} = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{C_t}{A_t\bar{N}_t} \right)^{-\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t\bar{N}_t} \right)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{P_t^K}{P_t} \right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}} \tag{1.18}$$

$$(xix).\;\; \frac{K_t}{A_{t-1}\bar{N}_{t-1}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \cdot GA_t \cdot GN \cdot \left(\frac{C_t}{A_t\bar{N}_t} \right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_t}{A_t\bar{N}_t} \right)^{\frac{\eta+1}{\eta+\alpha}} \left(\frac{P_t^K}{P_t} \right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \Delta_t \tag{1.19}$$

$$(xx).\;\; \frac{B_t^G}{P_tA_t\bar{N}_t} = \frac{R_t^G}{\Pi_t \cdot GA_t \cdot GN} \cdot \frac{B_{t-1}^G}{P_{t-1}A_{t-1}\bar{N}_{t-1}} - \left[\zeta_t^G + \zeta^F - \zeta_t^T \right] \left(\frac{Y_t}{A_t\bar{N}_t} \right) \tag{1.20}$$

$$(xxi).\;\; \lambda_t^{G,1} = \frac{1}{1 + \sum_{l=2}^{F} a^{B,l} \exp\left(\tilde{u}_t^{B,l}\right)},\;\; \lambda_t^{G,f} = \frac{a^{B,f} \exp\left(\tilde{u}_t^{B,f}\right)}{1 + \sum_{l=2}^{F} a^{B,l} \exp\left(\tilde{u}_t^{B,l}\right)},\;\; \forall f > 1 \tag{1.21}$$

$$(xxiii).\;\; \tilde{u}_t^{B,f} = \sum_{j=1}^{J} \tau_{fj}^B u_t^{B,j} \tag{1.22}$$

$$(xxiv).\;\; u_t^{B,j} = \rho_B u_{t-1}^{B,j} + \varepsilon_t^{B,j} \tag{1.23}$$

$$(xxv).\;\; B_t^{G,f} = \lambda_t^{G,f} B_t^G, \qquad \forall f = 1, \ldots, F \tag{1.24}$$

$$(xxvi).\;\; GA_t = \exp(\mu + \varepsilon_t^A) \tag{1.25}$$

$$(xxvii).\;\; \zeta_t^G = \frac{1}{1 + a^G \exp\left(-u_t^G\right)} \tag{1.26}$$

$$(xxviii).\;\; \zeta_t^T = \frac{1}{1 + a^T \exp\left(-u_t^T\right)} \tag{1.27}$$

$$(xxix).\;\; u_t^G = \rho_G \cdot u_{t-1}^G + \varepsilon_t^G \tag{1.28}$$

$$(xxx).\;\; u_t^T = \rho_T \cdot u_{t-1}^T + \varepsilon_t^T \tag{1.29}$$

1.2 Equilibrium Equations: Yield-Curve-Control Policy (YCC)

Summary of relevant equilibrium conditions:

$$(i).\ \frac{C_t}{A_t\bar{N}_t} = \left(1-\zeta_t^G\right)\left(\frac{Y_t}{A_t\bar{N}_t}\right) + \left(\frac{1-\delta}{GA_t \cdot GN}\right)\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right) - \left(\frac{K_{t+1}}{A_t\bar{N}_t}\right) \tag{1.30}$$

$$(ii).\ 1 = \beta\mathbb{E}_t\left[\frac{R_{t+1}^S}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\right] \tag{1.31}$$

$$(iii).\ -\left(\frac{\lambda_t^{CB,1}-\lambda_t^{G,1}}{\lambda_t^{CB,1}-\lambda_t^{HB,1}}\right)\cdot\frac{B_t^G}{A_t\bar{N}_tP_t} = \gamma_L\cdot\left(1+\zeta^F\right)\cdot\left(\frac{1-\lambda_t^K}{\lambda_t^K}\right)\left(\frac{Y_t}{A_t\bar{N}_t}\right) \tag{1.32}$$

$$(iv).\ \lambda_t^{CB,f} = \frac{\lambda_t^{HB,f}\cdot\left(1-\sum_{i\neq\{f,1\}}\lambda_t^{CB,i}-\lambda_t^{G,1}\right)-\lambda_t^{G,f}\cdot\left(1-\sum_{i\neq\{f,1\}}\lambda_t^{CB,i}-\lambda_t^{HB,1}\right)}{\left(\lambda_t^{HB,1}+\lambda_t^{HB,f}\right)-\left(\lambda_t^{G,1}+\lambda_t^{G,f}\right)},\qquad f=2,\ldots,F \tag{1.33}$$

$$(v).\ YD_t^1 = \max\left\{YD_t^{1*},1\right\} \tag{1.34}$$

$$(vi).\ YD_t^{1*} = \overline{YD}^1\cdot\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi^1}\left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_y^1}\cdot\exp\left(\tilde{\varepsilon}_t^{YD^1}\right) \tag{1.35}$$

$$(vii).\ YD_t^{YCC,f} = \overline{YD}^{YCC,f}\cdot\left(\frac{YD_t^{CP,f}}{\overline{YD}^{CP,f}}\right)^{\gamma_{CP}^f}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi^f}\left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_y^f}\cdot\exp\left(\tilde{\varepsilon}_t^{YD^f}\right)\right]^{1-\gamma_{CP}^f},\quad f\geq 2 \tag{1.36}$$

$$(viii).\ \lambda_t^{HB,f} = \left(\frac{\mathbb{E}_t\left[\frac{\beta z_t^f}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\frac{\left(YD_{t+1}^{f-1}\right)^{-(f-1)}}{\left(YD_t^f\right)^{-f}}\right]}{\Phi_t^B}\right)^{\kappa_B},\quad\forall f \tag{1.37}$$

$$(ix).\ \Phi_t^B = \left[\sum_{j=1}^{F}\mathbb{E}_t\left[\frac{\beta z_t^j}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\frac{\left(YD_{t+1}^{j-1}\right)^{-(j-1)}}{\left(YD_t^j\right)^{-j}}\right]^{\kappa_B}\right]^{\frac{1}{\kappa_B}} \tag{1.38}$$

$$(x).\ \lambda_t^K = \left(\frac{z_t^K\cdot\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]}{\Phi_t^S}\right)^{\kappa_S} \tag{1.39}$$

$$(xi).\ \ \Phi_t^S = \left[\left(\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{HB}\right]\right)^{\kappa_S} + \left(z_t^K\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{K}\right]\right)^{\kappa_S}\right]^{\frac{1}{\kappa_S}} \tag{1.40}$$

$$(xii).\ \ R_t^j = \sum_{f=0}^{F-1}\lambda_{t-1}^{j,f+1}\frac{\left(YD_t^f\right)^{-f}}{\left(YD_{t-1}^{f+1}\right)^{-(f+1)}} \qquad j\in\{HB,G,CB\} \tag{1.41}$$

$$(xiii).\ \ R_t^S = \left(1-\lambda_{t-1}^K\right)R_t^{HB} + \lambda_{t-1}^K R_t^K \tag{1.42}$$

$$(xiv).\ \ 1 = \mathbb{E}_t\left[Q_{t,t+1}\Pi_{t+1}\left[(1-\delta)+\frac{P_{t+1}^K}{P_{t+1}}\right]\right] \tag{1.43}$$

$$(xv).\ \ F_t = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right)\left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}\left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}}\left(\frac{P_t^K}{P_t}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}F_{t+1}\right] \tag{1.44}$$

$$(xvi).\ \ H_t = \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-1}\frac{Y_t}{A_t\bar{N}_t}\left[1-\gamma_L\cdot\left(\widetilde{R}_{t+1}^K-1\right)\right] + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon-1}H_{t+1}\right] \tag{1.45}$$

$$(xvii).\ \ \frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} \tag{1.46}$$

$$(xviii).\ \ \Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\Delta_{t-1} \tag{1.47}$$

$$(xix).\ \ \frac{N_t}{\bar{N}_t} = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)}\left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta}{\eta+\alpha}\right)}\left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\left(\frac{\eta}{\eta+\alpha}\right)}\left(\frac{P_t^K}{P_t}\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)}\Delta_t^{\frac{\eta}{\eta+1}} \tag{1.48}$$

$$(xx).\ \ \frac{K_t}{A_{t-1}\bar{N}_{t-1}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\cdot GA_t\cdot GN\cdot\left(\frac{C_t}{A_t\bar{N}_t}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}}\left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}}\left(\frac{P_t^K}{P_t}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)}\Delta_t \tag{1.49}$$

$$(xxi).\ \ \frac{B_t^G}{P_tA_t\bar{N}_t} = \frac{R_t^G}{\Pi_t\cdot GA_t\cdot GN}\frac{B_{t-1}^G}{P_{t-1}A_{t-1}\bar{N}_{t-1}} - \left[\zeta_t^G+\zeta^F-\zeta_t^T\right]\left(\frac{Y_t}{A_t\bar{N}_t}\right) \tag{1.50}$$

$$(xxii).\ \ \tilde{\varepsilon}_t^{YD,f} = \sum_{l=1}^{L}\tau_{fl}^{YD}\varepsilon_t^{YD,l} \tag{1.51}$$

1.3 Summary of Conventional Policy Linearized Equations

Those are the essential equations to solve the model, other variables can be found on the equations above.

$$(i).\ \hat{y}_t = \mathbb{E}_t\Big[\hat{y}_{t+1} + \Psi^{46}\cdot\hat{\pi}_{t+1} - \Psi^{47}\cdot\overrightarrow{\hat{z}}_t - \Psi^{48}\cdot\hat{z}_t^K - \Psi^{49}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{50}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right]$$

$$- \Psi^{51}\cdot\hat{r}_{t+1}^K - \Psi^{52}\cdot(\hat{k}_t - \hat{\varepsilon}_t^A) + \Psi^{53}\cdot\hat{k}_{t+1} - \Psi^{54}\cdot\hat{k}_{t+2} + \Psi^{55}\cdot\hat{u}_t^G\Big]$$

$$(ii).\ \overrightarrow{\hat{yd}_t} = \Theta^{14}\cdot\hat{b}_t^G - \Theta^{15}\cdot\hat{y}_t + \Theta^{16}\cdot\hat{r}_{t+1}^K - \Theta^{17}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] - \Theta^{18}\cdot\overrightarrow{\hat{z}_t} + \Theta^{19}\cdot\hat{z}_t^K + \Theta^{20}\cdot\overrightarrow{\hat{u}_t^B}$$

$$(iii).\ \hat{yd}_t^1 = \max\left\{\hat{yd}_t^{1*}, 0\right\}$$

$$(iv).\ \hat{yd}_t^{1*} = \gamma_\pi\hat{\pi}_t + \gamma_y\hat{y}_t + \tilde{\varepsilon}_t^{YD^1},\ \ \tilde{\varepsilon}_t^{YD^f} = \sum_{l=1}^{L}\tau_{f,l}^{YD}\varepsilon_t^{YD^l}$$

$$(v).\ \hat{r}_{t+1}^K = -\Psi^{37}\cdot\overrightarrow{\hat{z}_t} - \Psi^{38}\cdot\hat{z}_t^K - \Psi^{39}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{40}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{41}\cdot\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{42}\cdot\mathbb{E}_t\left[\hat{y}_{t+1}\right]$$

$$+ \Psi^{43}\cdot\hat{k}_{t+1} - \Psi^{44}\cdot\mathbb{E}_t\left[\hat{k}_{t+2}\right] - \Psi^{45}\cdot\hat{u}_t^G$$

$$(vi).\ \hat{b}_t^G = \frac{R^G}{\Pi\cdot GA\cdot GN}\cdot\left[\Psi^{G,4}\Xi\overrightarrow{\hat{u}_{t-1}^B} - \Psi^{G,5}\overrightarrow{\hat{yd}_t} + \Psi^{G,6}\overrightarrow{\hat{yd}_{t-1}} - \hat{\pi}_t - \hat{\varepsilon}_t^A + \hat{b}_{t-1}^G\right]$$

$$+\left(1 - \frac{R^G}{\Pi\cdot GA\cdot GN}\right)\left[\hat{y}_t + \left(\frac{\zeta^G}{\zeta^G + \zeta^F - \zeta^T}\right)\frac{a^G}{1+a^G}\hat{u}_t^G - \left(\frac{\zeta^T}{\zeta^G + \zeta^F - \zeta^T}\right)\frac{a^T}{1+a^T}\hat{u}_t^T\right]$$

$$(vii).\ \hat{f}_t = -\Psi^{16}\cdot\overrightarrow{\hat{z}_t} - \Psi^{17}\cdot\hat{z}_t^K - \Psi^{18}\cdot\left[\hat{k}_t - \hat{\varepsilon}_t^A\right] + \Psi^{19}\cdot\hat{y}_t - \Psi^{20}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{21}\cdot\hat{r}_{t+1}^K - \Psi^{22}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right]$$

$$+ \Psi^{23}\cdot\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{24}\cdot\mathbb{E}_t\left[\hat{f}_{t+1}\right]$$

$$(viii).\ \hat{h}_t = \Psi^{26}\cdot\overrightarrow{\hat{z}_t} + \Psi^{27}\cdot\hat{z}_t^K - \Psi^{28}\cdot\left[\hat{k}_t - \hat{\varepsilon}_t^A\right] + \Psi^{29}\cdot\hat{u}_t^G + \Psi^{30}\cdot\hat{y}_t + \Psi^{31}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{32}\cdot\hat{r}_{t+1}^K$$

$$+ \Psi^{33}\cdot\hat{k}_{t+1} + \Psi^{34}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{35}\cdot\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{36}\cdot\mathbb{E}_t\left[\hat{h}_{t+1}\right]$$

$$(ix).\ \hat{f}_t - \hat{h}_t = \left[1 + \epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\left(\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)\hat{\pi}_t$$

$$(x).\ u_t^{B,j} = \rho_B\cdot u_{t-1}^{B,j} + \varepsilon_t^{B,j}, \qquad \forall\ j = 1,\dots,J$$

$$(xi).\ u_t^G = \rho_G\cdot u_{t-1}^G + \varepsilon_t^G$$

$$(xii).\ u_t^T = \rho_T\cdot u_{t-1}^G + \varepsilon_t^T$$

1.4 Summary of Yield-Curve-Control Policy Linearized Equations

Those are the essential equation to solve the model, other variables can be found on equations above.

$$(i).\ \hat{y}_t = \mathbb{E}_t\Big[\hat{y}_{t+1} + \Psi^{46}\cdot\hat{\pi}_{t+1} - \Psi^{47}\cdot\overrightarrow{\hat{z}}_t - \Psi^{48}\cdot\hat{z}^K_t - \Psi^{49}\cdot\overrightarrow{\hat{yd}^{YCC}_t} - \Psi^{50}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}^{YCC}_{t+1}}\right]$$

$$- \Psi^{51}\cdot\hat{r}^K_{t+1} - \Psi^{52}\cdot(\hat{k}_t - \hat{\varepsilon}^A_t) + \Psi^{53}\cdot\hat{k}_{t+1} - \Psi^{54}\cdot\hat{k}_{t+2} + \Psi^{55}\cdot\hat{u}^G_t\Big]$$

$$(ii).\ \overrightarrow{\hat{yd}^{CP}_t} = \Theta^{14}\cdot\hat{b}^G_t - \Theta^{15}\cdot\hat{y}_t + \Theta^{16}\cdot\hat{r}^K_{t+1} - \Theta^{17}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}^{YCC}_{t+1}}\right] - \Theta^{18}\cdot\overrightarrow{\hat{z}_t} + \Theta^{19}\cdot\hat{z}^K_t + \Theta^{20}\cdot\overrightarrow{\hat{u}^B_t}$$

$$(iii).\ \hat{yd}^{YCC,1}_t = \max\left\{\hat{yd}^{1*}_t, 0\right\} = \hat{yd}^{CP,1}_t$$

$$(iv).\ \hat{yd}^{1*}_t = \gamma_\pi\hat{\pi}_t + \gamma_y\hat{y}_t + \tilde{\varepsilon}^{YD^1}_t,\ \ \tilde{\varepsilon}^{YD^1}_t = \sum_{l=1}^{L}\tau^{YD}_{1,l}\varepsilon^{YD^l}_t,$$

$$(v).\ \hat{yd}^{YCC,f}_t = \gamma^f_{CP}\hat{yd}^{CP,f}_t + \left(1-\gamma^f_{CP}\right)\left[\gamma^f_\pi\hat{\pi}_t + \gamma^f_y\hat{y}_t + \tilde{\varepsilon}^{YD^f}_t\right],\ \ \tilde{\varepsilon}^{YD^f}_t = \sum_{l=1}^{L}\tau^{YD}_{f,l}\varepsilon^{YD^l}_t,\quad f\geq 2$$

$$(vi).\ \hat{r}^K_{t+1} = -\Psi^{37}\cdot\overrightarrow{\hat{z}_t} - \Psi^{38}\cdot\hat{z}^K_t - \Psi^{39}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{40}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{41}\cdot\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{42}\cdot\mathbb{E}_t\left[\hat{y}_{t+1}\right]$$

$$+ \Psi^{43}\cdot\hat{k}_{t+1} - \Psi^{44}\cdot\mathbb{E}_t\left[\hat{k}_{t+2}\right] - \Psi^{45}\cdot\hat{u}^G_t$$

$$(vii).\ \hat{b}^G_t = \frac{R^G}{\Pi\cdot GA\cdot GN}\cdot\left[\Psi^{G,4}\Xi\overrightarrow{\hat{u}^B_{t-1}} - \Psi^{G,5}\overrightarrow{\hat{yd}^{YCC}_t} + \Psi^{G,6}\overrightarrow{\hat{yd}^{YCC}_{t-1}} - \hat{\pi}_t - \hat{\varepsilon}^A_t + \hat{b}^G_{t-1}\right]$$

$$+\left(1-\frac{R^G}{\Pi\cdot GA\cdot GN}\right)\left[\hat{y}_t + \left(\frac{\zeta^G}{\zeta^G+\zeta^F-\zeta^T}\right)\frac{a^G}{1+a^G}\hat{u}^G_t - \left(\frac{\zeta^T}{\zeta^G+\zeta^F-\zeta^T}\right)\frac{a^T}{1+a^T}\hat{u}^T_t\right]$$

$$(viii).\ \hat{f}_t = -\Psi^{16}\cdot\overrightarrow{\hat{z}_t} - \Psi^{17}\cdot\hat{z}^K_t - \Psi^{18}\cdot\left[\hat{k}_t - \hat{\varepsilon}^A_t\right] + \Psi^{19}\cdot\hat{y}_t - \Psi^{20}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{21}\cdot\hat{r}^K_{t+1} - \Psi^{22}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right]$$

$$+ \Psi^{23}\cdot\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{24}\cdot\mathbb{E}_t\left[\hat{f}_{t+1}\right]$$

$$(ix).\ \hat{h}_t = \Psi^{26}\cdot\overrightarrow{\hat{z}_t} + \Psi^{27}\cdot\hat{z}^K_t - \Psi^{28}\cdot\left[\hat{k}_t - \hat{\varepsilon}^A_t\right] + \Psi^{29}\cdot\hat{u}^G_t + \Psi^{30}\cdot\hat{y}_t + \Psi^{31}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{32}\cdot\hat{r}^K_{t+1} + \Psi^{33}\cdot\hat{k}_{t+1}$$

$$+ \Psi^{34}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{35}\cdot\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{36}\cdot\mathbb{E}_t\left[\hat{h}_{t+1}\right]$$

$$(x).\ \hat{f}_t - \hat{h}_t = \left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\left(\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)\hat{\pi}_t$$

$$(xi).\ u^{B,j}_t = \rho_B\cdot u^{B,j}_{t-1} + \varepsilon^{B,j}_t,\qquad \forall\, j=1,\ldots,J$$

$$(xii).\ u^G_t = \rho_G\cdot u^G_{t-1} + \varepsilon^G_t$$

$$(xiii).\ u^T_t = \rho_T\cdot u^G_{t-1} + \varepsilon^T_t$$

2 Additional Results: Steady-State Comparative Statics

Comparative statics with κ_B Figure 2.1 demonstrates the behavior of the steady-state yield curve as κ_B increases, as explained in Section 2.1.1. With $R^K > R^{HB}$ at the steady state and low levels of z^f for high f in Figure 1.1, an increase in κ_B lowers the household's demand for long-term bonds (as markets are more competitive with higher κ_B), pushing up long-term yields. When $\kappa_B \to \infty$, we revert to the expectations hypothesis case, resulting in a flat yield curve in the steady-state.

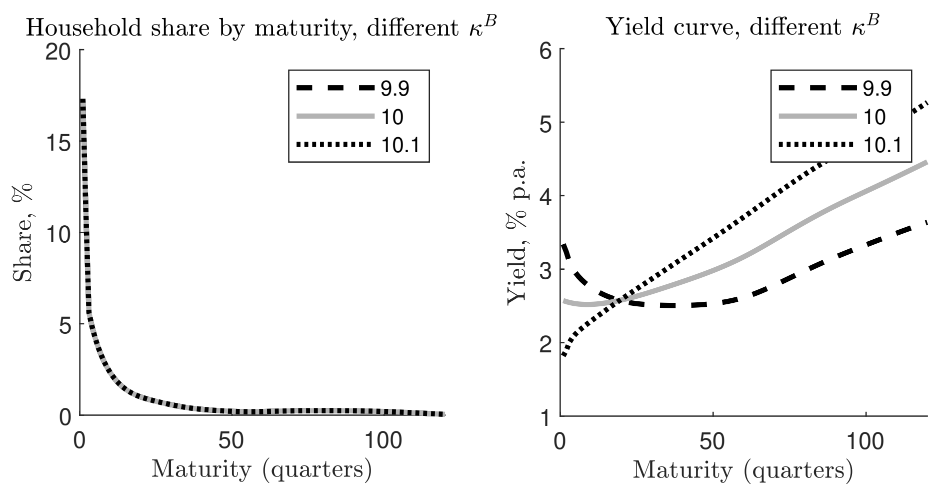


Figure 2.1: Variations in κ_B (i.e., scale parameter): as $\kappa_B \to \infty$, the model converges to the expectation hypothesis, wherein all discounted expected returns become equalized. With current $R^K > R^{HB}$ at the steady state and low levels of z^f for high f in Figure 1.1, higher κ_B lowers the household's demand for long-term bonds, pushing up their yields.

Comparative statics with z^K Figure 2.2 illustrate comparative statics with respect to the scale parameter z^K, which is incorporated into the savings allocation between the Treasury and loan markets (i.e., (6)). Given the calibrated $\{z^f\}_{f=1}^F$ and for $z^K \in [0.8, 1.3]$, a higher z^K suggests that the household is more inclined to provide loans to firms rather than invest in the bond market, raising λ^K. This leads to higher capital, output, and consumption in the steady-state allocation. As households increase their loan investment, the average marginal propensity to consume and the equilibrium loan rate R^K decline, causing the entire yield curve to shift downward due to the household's endogenous portfolio reallocation —paradoxically raising credit spreads. Falling R^G lowers the government bond share with respect to GDP.

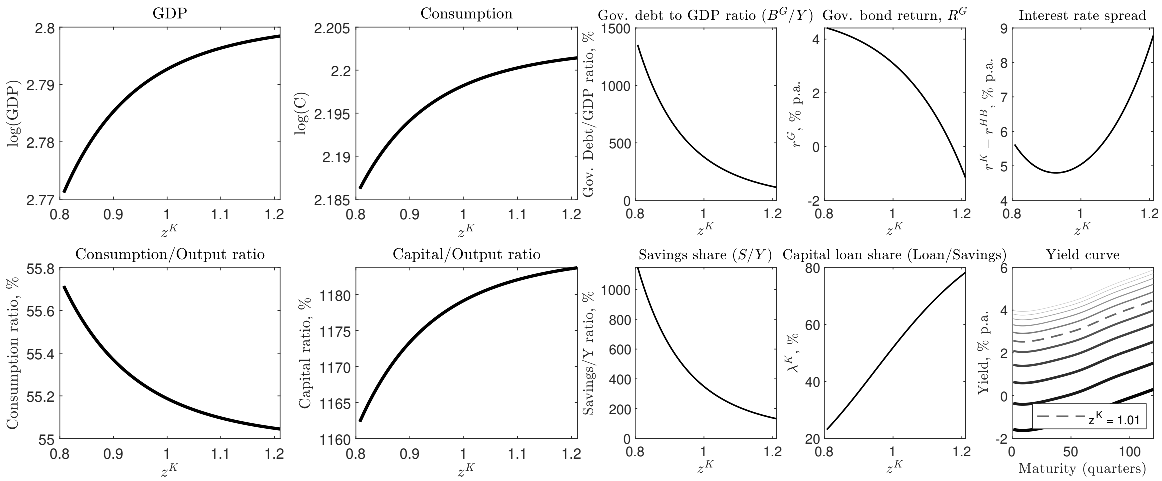


Figure 2.2: Variations in z^K

Comparative statics with κ_S Figure 2.3 depicts comparative statics of the shape parameter κ_S, which appears in the same savings allocation condition between bond and loan markets (i.e., equation (6)). Given the calibrated $\{z^f\}_{f=1}^{F}$ and z^K values, and for $\kappa_S \in [0, 25]$, a higher κ_S raises λ^K, as $R^K > R^{HB}$ at the steady state and higher κ_S implies that the market is more competitive. This results in a lower loan rate R^K, higher capital demand (as firms face lower interest costs), increased output and consumption while reducing the average marginal propensity to consume. Credit spreads widen, with a lower R^K depressing the government's effective bond return R^G and the entire yield curve even more. Consequently, the government's debt-to-GDP ratio falls.

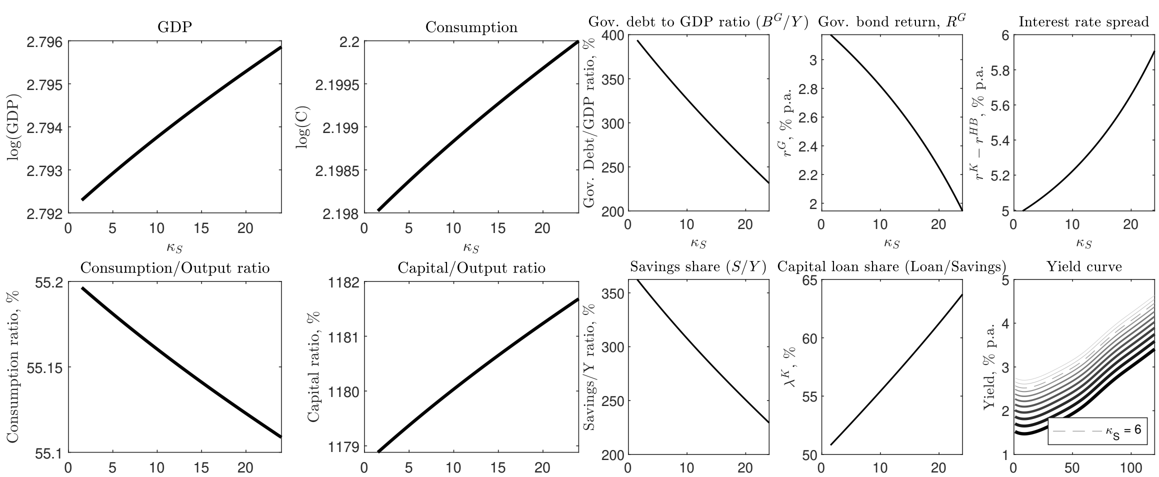


Figure 2.3: Variations in κ^S

3 Additional Results: Impulse-Responses

3.1 Without ZLB

Technology shock, ε_t^A: Figure 3.1 displays the impulse-responses to a ε_t^A shock. A positive shock in technology growth GA_t yields similar effects as documented in prior literature under conventional policy,[10] with output increasing[11] as inflation decreases. Under the yield-curve-control regime, the normalized output decreases less: since inflation falls, bond yields shift downwards, boosting consumption and reducing both the return on capital and the wage.

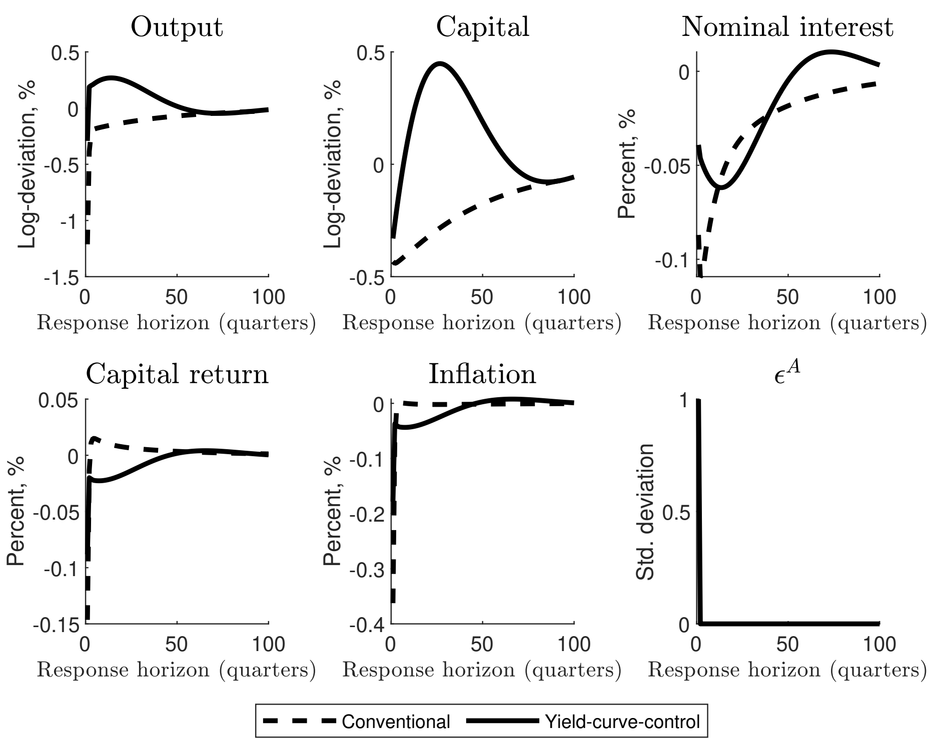


Figure 3.1: Impulse response to a ε_t^A shock: A positive technology growth shock generates effects in line with the existing literature, resulting in an increase in output and a decline in inflation. As inflation decreases, all yields undergo a downward shift, which in turn lowers both the capital return and wage relative to the conventional scenario.

[10]For effects of technology shocks in a canonical New-Keynesian model, see Ireland (2004).
[11]Even if the 'normalized' output drops under our calibration, the output level rises.

Monetary policy shock, $\varepsilon_t^{YD^1}$: Figure 3.2 presents the impulse-response to a $\varepsilon_t^{YD^1}$ shock. Under the conventional policy, a contractionary monetary policy shock paradoxically results in an increase in output, inflation, and capital. One potential channel is from increases in the household's interest incomes from higher rates on both bonds and loans. The yield-curve-control policy insulates the economy as before: in response to the shock, the central bank pushes down the entire yield curve, preventing input prices (i.e., loan rates and wages) from rising too high, and thereby mitigating changes in output and inflation.

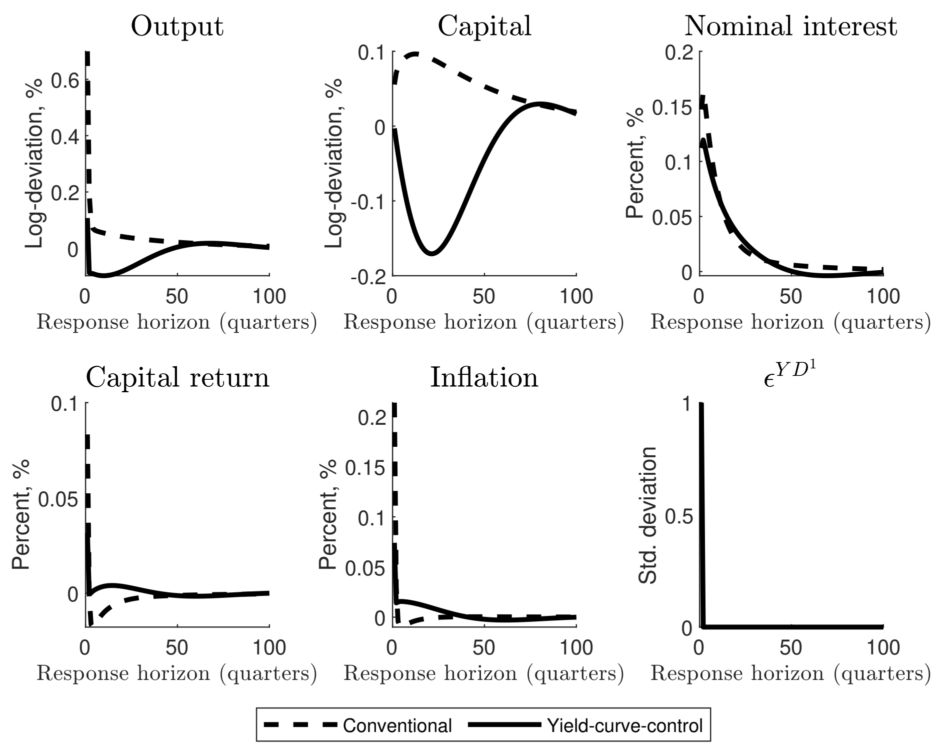


Figure 3.2: Impulse response to a $\varepsilon_t^{YD^1}$ shock: A conventional contractionary monetary policy shock paradoxically leads to increases in output, inflation, and capital due to higher amounts of interest incomes that households earn from bonds and loans. The yield-curve-control policy insulates the economy against the shock through the central bank's purchases of long-term bonds.

3.2 With ZLB

With mixed policy, z_t^K: In Figure 3.3, the mixed policy actually features the longest ZLB spell.

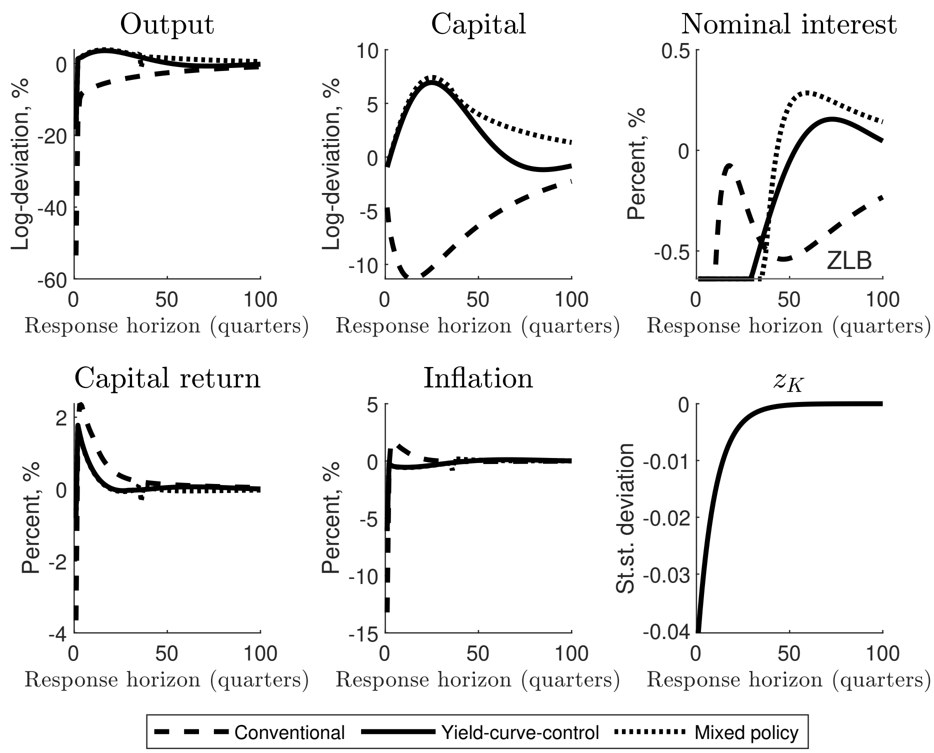


Figure 3.3: Impulse-response to z^K shock with ZLB: with mixed policy